INDEX DECOMPOSITION ANALYSIS:

SOME METHODOLOGICAL ISSUES

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NATIONAL UNIVERSITY OF SINGAPORE

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DECLARATION

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

MU AORAN
14 AUGUST, 2012

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SUMMARY

The economic and social impacts of high crude oil prices, the security of energy supplies, and concerns over global warming have put pressure on many countries to implement energy efficiency and conservation programs. How to track energy efficiency and to evaluate the performance of energy efficiency and conservation programs is an important issue for energy policy analysts and decision makers. Index decomposition analysis (IDA) has been a popular tool for tracking and monitoring economy-wide or sectoral energy efficiency and analyzing the impacts of factors influencing the change of various energy-related aggregate indices or indicators. IDA has been investigated in many research studies and has been applied in many international and national energy efficiency accounting systems to track energy efficiency trends. Due to the importance of IDA in energy analysis, this thesis presents a comprehensive review of IDA and investigates some related methodological issues.

This thesis is divided into four parts. In the first part, we present a comprehensive literature review of energy-related IDA studies to provide an overview of the development of IDA and to situate current IDA studies, which also helps to identify the research gaps and explain the motivation for the research topics discussed in this thesis.

In the second part, we systematically study the linkages and differences between IDA and index number problems (INP), which is the theoretical foundation of the development of IDA. In addition, new tests are derived from

INP and a summary of criteria to evaluate IDA methods is provided to help researchers in the understanding and application of IDA methods corresponding to different situations and data sources.

In the third part, we focus on methodological issues in IDA methods. The relationship between Laspeyres-based IDA methods and the Shapley value in game theory is formalized. Properties and linkages among additive Divisia-based IDA methods are discussed. In addition, recommendations for IDA method selection are discussed, and it is concluded that the Logarithmic Mean Divisia Index I (LMDI I) method is the preferred Divisia –based IDA method.

Finally, one important IDA methodological issue, treatment of time problems, is studied. Chaining and non-chaining are two approaches to treating time in IDA and there is still no consensus among researchers about the preferred choice. A comprehensive comparison of the advantages and disadvantages of these two approaches is presented.

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LIST OF ABBREVIATIONS

AMDI	Arithmetic Mean Divisia Index
AWD	Adaptive Weighing Divisia
COLI	Cost-of-Living Index
CPI	Consumer Price Index
EERE	Office of Energy Efficiency and Renewable Energy
GDP	Gross Domestic Product
GHG	Greenhouse Gas
GNP	Gross National Product
IDA	Index Decomposition Analysis
INP	Index Number Problem
LAS-PDM1	Laspeyres-based Parametric Divisia Method 1
LMDI I	Logarithmic Mean Divisia Index I
LMDI II	Logarithmic Mean Divisia Index II
M-E	Marshall-Edgeworth
Mtoe	Million Tonnes of Oil Equivalent
OECD	Organisation for Economic Co-operation and Development
OEE	Office of Energy Efficiency

PDM	Parametric Divisia Method
RL	Refined Laspeyres
SDA	Structural Decomposition Analysis
S/S	Shapley/Sun Method
TL	Traditional Laspeyres

LIST OF NOTATIONS

E : Total energy consumption ($E=\sum_{j} E_j$)

C: Total CO_2 emission ($C=\sum_{j} C_j$)

Y : Total activity ($Y=\sum_{j} Y_j$)

E_j : Energy consumption in sector j

C_j : CO_2 emission in sector j

Y_j : Activity of sector j

S_j : Activity share of sector j ($S_j = Y_j / Y$)

I : Aggregate energy intensity ($I = E/Y$)

I_j : Energy intensity for sector j ($I_j = E_j / Y_j$)

e_j : Energy emission factor for sector j ($e_j = C_j / E_j$)

ΔC_{tot} : Actual CO_2 emission change in difference between year 0 and year T ($\Delta C_{tot} = C^T - C^0$)

ΔC_{act} : Estimate of the change in CO_2 emission due to the change in the overall level of activity in additive form

$\Delta C_{act'}$: Estimate of the change in CO_2 emission due to the change in the sectoral level of activity in additive form

ΔC_{str} : Estimate of the change in CO_2 emission due to the change in the structure in additive form

ΔC_{int} : Estimate of the change in CO_2 emission due to the change in the sectoral energy intensity in additive form

$\Delta C_{emf'}$: Estimate of the change in CO_2 emission due to the change in the CO_2 emission factor without eliminating energy mix effect in additive form

ΔC_{emf} : Estimate of the change in CO_2 emission due to the change in the CO_2 emission factor by fuel in additive form

ΔC_{mix} : Estimate of the change in CO_2 emission due to the change in the energy mix in additive form

ΔC_{rsd} : Estimated residual term in additive form

ΔE_{tot} : Actual energy consumption change in difference between year 0 and year T ($\Delta E_{tot} = E^T - E^0$)

ΔE_{act} : Estimate of the change in energy consumption due to the change in the overall activity in additive form

$\Delta E_{act'}$: Estimate of the change in energy consumption due to the change in the sectoral level of activity in additive form

ΔE_{str} : Estimate of the change in energy consumption due to the change in the structure in additive form

ΔE_{int} : Estimate of the change in energy consumption due to the change in the sectoral intensity in additive form

ΔE_{rsd} : Estimated residual term in additive form ($\Delta E_{rsd} = \Delta E_{tot} - (\Delta E_{act} + \Delta E_{str} + \Delta E_{int})$)

ΔI_{tot} : Actual energy intensity change in difference between year 0 and year T ($\Delta I_{tot} = I^T - I^0$)

ΔI_{str} : Estimate of the change in energy intensity due to the change of structure effect in additive form

ΔI_{int} : Estimate of the change in energy intensity due to the change of intensity effect in additive form

ΔI_{rsd}: Estimated residual term in additive form ($\Delta I_{rsd} = \Delta I_{tot} - \Delta I_{str} - \Delta I_{int}$)

V: Energy-related aggregate

V_j: Energy-related aggregate for sector j

$\Delta V_{tot}^{0,T}$: Energy-related aggregate in difference between year 0 and year T ($\Delta V_{tot}^{0,T} = V^T - V^0$)

$\Delta V_{x_i}^{0,T}$: Estimate of the change in energy-related aggregate V due to the change in factor x_i in additive form between year 0 and year T

$\Delta V_{rsd}^{0,T}$: Estimated residual term in additive form ($\Delta V_{rsd}^{0,T} = \Delta V_{tot}^{0,T} - \sum_{i=1}^{n} \Delta V_{x_i}^{0,T}$) between year 0 and year T

$\Delta V_{tot}^{t,t+1}$: Energy-related aggregate in difference between year t and year t+1 ($\Delta V_{tot}^{t,t+1} = V^{t+1} - V^t$)

$\Delta V_{x_i}^{t,t+1}$: Estimate of the change in energy-related aggregate V due to the change in factor x_i in additive form between year t and year t+1

$\Delta V_{rsd}^{t,t+1}$: Estimated residual term in additive form ($\Delta V_{rsd}^{t,t+1} = \Delta V_{tot}^{t,t+1} - \sum_{i=1}^{n} \Delta V_{x_i}^{t,t+1}$) between year t and year t+1

$D_{tot}^{0,T}$: Ratio change of energy-related aggregate between year 0 and year T ($D_{tot}^{0,T} = V^T / V^0$)

$D_{x_i}^{0,T}$: Estimate of the change in energy-related aggregate V due to the change in factor x_i in multiplicative form between year 0 and year T

$D_{rsd}^{0,T}$: Estimated residual term in multiplicative form ($D_{rsd}^{0,T} = D_{tot}^{0,T} / \prod_{i} D_{x_i}^{0,T}$) between year 0 and year T

$D_{tot}^{t,t+1}$: Ratio change of energy-related aggregate between year t and year t+1 ($D_{tot}^{t,t+1} = V^{t+1} / V^t$)

$D_{x_i}^{t,t+1}$: Estimate of the change in energy-related aggregate V due to the change in factor x_i in multiplicative form between year t and year t+1

$D_{rsd}^{t,t+1}$: Estimated residual term in multiplicative form ($D_{rsd}^{t,t+1}=D_{tot}^{t,t+1}/\prod_i D_{x_i}^{t,t+1}$) between year t and year t+1

p_i: Price for commodity i

q_i: Quantity for commodity i

v_i: Value for commodity i

$p \equiv (p_1,\ldots,p_N)$: Vector of price

$q \equiv (q_1,\ldots,q_N)$: Vector of quantity

$v \equiv (v_1,\ldots,v_N)$: Vector of value

$P(p^T,q^T,p^0,q^0)$: Price index for period T relative to period 0

$Q(p^T,q^T,p^0,q^0)$: Quantity index for period T relative to period 0

$\mathcal{P}(p^T,q^T,p^0,q^0)$: Price indicator for period T relative to period 0

$\mathcal{Q}(p^T,q^T,p^0,q^0)$: Quantity indicator for period T relative to period 0

CHAPTER 1: INTRODUCTION

This thesis contributes to some methodological issues of IDA, with its applications in energy studies. In this introductory chapter, some background information is presented and some concepts related to IDA are introduced. This is followed by an introduction to methodological issues in IDA. Finally, the scope and structure of the thesis are provided.

1.1 IDA

The economic and social impacts of high oil prices, the security of energy supplies and global warming problems have put pressure on most countries to improve energy efficiency. Energy efficiency improvement helps reduce growth in energy demand, enhance energy security and moderate the impacts of energy on the environment. In many countries, energy efficiency and conservation programs have been implemented economy-wide, covering all major energy-consuming sectors. In some countries, targets and actions are specified with clear accountability for delivery.

Technical issues that arise from these initiatives include: how to quantify the impacts of various factors influencing energy consumption and CO_2 emissions, and how to define economy-wide energy efficiency and how to track its performance over time. In the 1970s and early 1980s, the ratio of total national primary energy consumption to GDP (or GNP) was the main indicator of energy efficiency due mainly to its simplicity and to the paucity of energy consumption data. This approach tends to have limited explanatory power, since it does not isolate changes in economic structure and other factors which affect changes in energy efficiency. IDA is a technique used to quantify the effects of factors influencing the changes of energy-related aggregate indices and indicators. After isolating and removing all the other effects, the energy intensity effect is usually taken as a proxy for energy efficiency. Therefore, IDA has been a popular tool to track energy efficiency trends and to analyze the impacts of factors influencing changes in energy consumption and CO_2 emissions. It assists in evaluating energy efficiency and conservation programs and helps energy policy analysts and decision makers formulate and evaluate energy policy and targets.

IDA can be conducted either by the additive decomposition approach or multiplicative decomposition approach. In additive decomposition analysis, changes in an energy-related aggregate are measured as a difference and the decomposition results are given in a physical unit, which is the same as the physical unit of the energy-related aggregate. In multiplicative decomposition analysis, changes in an energy-related aggregate are measured as a ratio and the decomposition results are expressed as a dimensionless index.

A simple example is given below showing additive and multiplicative decomposition approaches. Assume that the energy consumption of the industrial sector in a country changes from 20 million tonnes of oil equivalent (Mtoe) to 30 Mtoe from year 0 to year T. In additive decomposition, we study how the factors contribute to a 10 Mtoe change (a difference of 30 Mtoe in year T and 20 Mtoe in year 0) by expressing the factor effects in a physical unit (Mtoe). In multiplicative decomposition, we study how the factors contribute to the 1.5 ratio change (the ratio of 30 Mtoe to 20 Mtoe) by expressing the factor effects in a dimensionless index.

1.2 IDA and Economic Theory

In economic theory, price indices study the changes of the general level of price through time, while the quantity indices study the changes of quantities of goods and services (so-called real developments) through time. The product of price and quantity is the expenditure value. Expenditure value change of economic flows can be decomposed into price and quantity indices to study how changes in price and quantity levels contribute to changes in aggregate commodity consumption. The main research areas of INP include the study of "purchasing power of money through time", "cost of living index", "consumption deflator", etc.

Boyd et al. (1988) first point out the relationships between IDA and INP. The authors comment that the problem of disaggregating changes of energy intensity at the aggregate level into their component parts is analogous to INP in economics. They also suggest that the IDA problem could borrow ideas from the index number literature and approach for some guidance on both methods and properties. Since then, many IDA methods and tests have been derived from the INP. Examples are the AMDI, LMDI I method, factor-reversal test, and time-reversal test.

1.3 IDA Methods

Since IDA was first introduced in the late 1970s to study the impact of changes in product mix on industrial energy demand, many IDA methods have been developed. Ang (2004) classifies the IDA methods into Laspeyres-based and Divisia-based methods as shown in Figure 1-1.

The decomposition formulae used by researchers prior to the mid-1980s are straightforward and intuitive. The impact of structure change was derived from the difference between the aggregate energy intensity in the target year with sectoral energy intensities for all industrial sectors remaining at their base year and the aggregate energy intensity in the base year. This decomposition method is similar to the Laspeyres price and quantity index (proposed by Laspeyres, 1871). The basic idea of the Laspeyres index is to isolate the impact of a pre-defined factor from the change of an aggregate indicator by changing this factor while holding all the other factors unchanged.

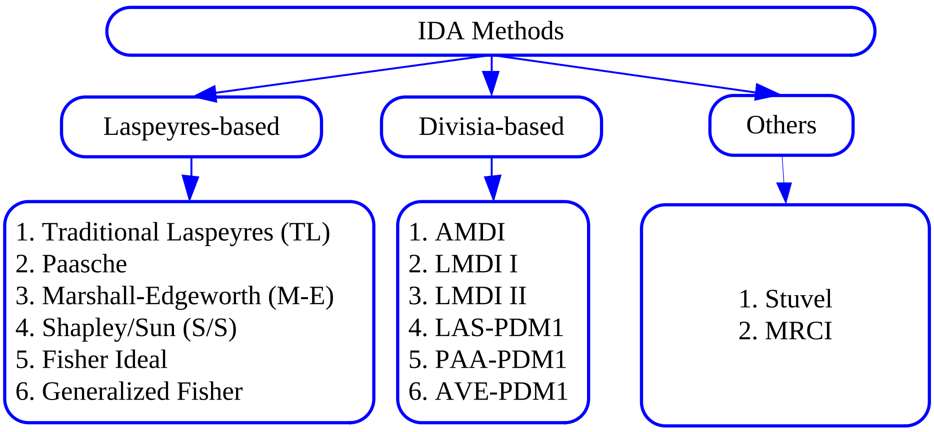

Figure 1-1. Classification of IDA methods

Laspeyres decomposition analysis leaves a residual term, which is hard to explain. Some researchers refined the traditional Laspeyres (TL) method and derived several “refined Laspeyres methods” without residuals. Sun (1998) proposes a method, in which the residual is distributed equally among the main effects based on the “jointly created and equally distributed” principle.

The Laspeyres-based category includes methods such as the TL method (base year weights), Paasche (terminal year weights), Marshall-Edgeworth (M-E) (mean of base and terminal year weights), and Refined Laspeyres (RL) methods like Shapley/Sun (S/S) method for additive analysis. In multiplicative approach, the Laspeyres-based category includes the Fisher method and generalized Fisher method.

The Divisia index is an integral index number developed by Divisia (1925). Boyd et al. (1987) apply the Divisia index approach in studying US industrial energy consumption. Since then, Divisia-based IDA methods have been widely acknowledged in decomposition of energy-related indicators. Divisia-based methods in this classification include integral IDA which is similar the Divisia index, i.e. all logarithmic mean methods and integral methods. The main idea of the Divisia decomposition method is to isolate the impact of a certain factor by taking the integration from period 0 to period T and assigning an appropriate weight for this factor under some assumptions of the integral path of the factor.

The popular Divisia-based IDA methods include Arithmetic Mean Divisia Index (AMDI), Logarithmic Mean Divisia Index I (LMDI I) and II (LMDI II), Laspeyres-based parametric Divisia method 1 (LAS-PDM1), Paasche-based parametric Divisia method 1 (PAA-PDM1), simple average parametric Divisia method 1 (AVE-PDM1) and some other Divisia-based methods.

In addition to Laspeyres-based methods and Divisia-based methods, other IDA methods, classified into "Others", include the Stuvel Index and the Mean-Rate-of-Change Index (MRCI). Both methods are seldom used in IDA since the Stuvel Index can handle only two contributing factors and the MRCI applies to only the additive analysis.

1.4 Treatment of Time

Chaining and non-chaining are two different indexing approaches in energy-related decomposition analysis. If a decomposition analysis is conducted over a time period consisting of a certain number of years using yearly data, say from year 0 to year *T*, we could conduct decomposition based only on the data for the starting year 0 and the ending year *T* without using the data in the intervening years. Alternatively, we could carry out decomposition using the data for every two consecutive years in the time series, i.e. years 0 and 1, 1 and 2, and so on till *T*-1 and *T*. A total of *T* sets of decomposition results can be obtained which can then be "chained" to give the results for the whole time period. The former is referred to as the "non-chaining" while the latter the "chaining" approach. When data are available for only two years which are not consecutive, non-chaining analysis is the only choice available to the analyst.

In IDA, the terminology for chaining and non-chaining is not consistent in different IDA studies. Some examples are given as follows. Ang and Lee (1994), and Liu et al. (2007) use the terms "time series (i.e. yearly) decomposition" and "period-wise decomposition"; Greening et al. (1997), and Bataille and Nyboer (2005) use "rolling base year decomposition" and "fixed base year decomposition"; and Ang (2004), and Ang and Liu (2007a) use "chaining decomposition" and "non-chaining decomposition". In this thesis, we opt to use "chaining decomposition" and "non-chaining decomposition" to keep the terminology consistent with that used in the index number literature.

1.5 Scope and Structure of the Thesis

This thesis focuses on some methodological issues of IDA and Figure 1-2 highlights the scope and structure of the thesis.

Chapter 2 presents a literature survey of IDA. Ang and Zhang (2000) conduct a survey of decomposition analysis in energy studies and list a total of 124 journal and conference papers from 1978 to 2000. Among them, 109 papers are related to IDA and 15 papers are about structure decomposition analysis (SDA). Since then, the number of studies has increased and there have been important new developments in both methodological and application aspects. However, there is still a lack of up-dated survey for IDA. In Chapter 2, we summarize the new developments of IDA in the last ten years to bring the survey up to date. Decomposition methods are classified using a new framework and more comprehensive information is provided.

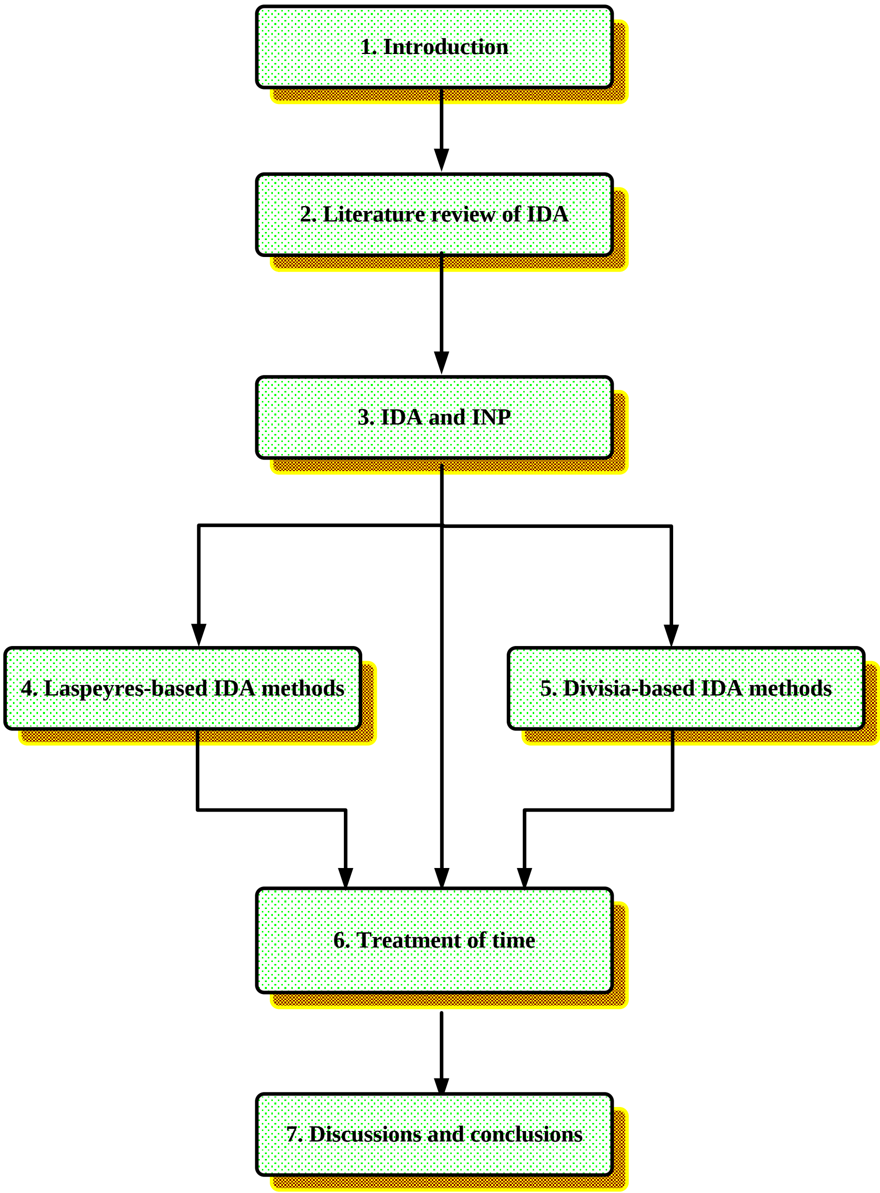

Figure 1-2. Structure of the thesis

Chapter 3 explores linkages and differences between IDA and INP. IDA and INP are closely related in terms of both methods and properties. Although IDA has been developed for more than 30 years, there is still a lack of studies which systematically explore the theoretical foundations of IDA from the viewpoint of INP in economics. In this chapter, we extend the studies of Boyd et al. (1988) and Liu and Ang (2003) to discuss the linkages and differences between IDA and INP. In addition, we summarize the existing tests to evaluate IDA methods, identify the problems of tests in IDA studies and introduce three new tests from INP to better understand whether a method is effective in performing decomposition analysis. In IDA, different methods have different formulae, which lead to different results. As a result, method selection for a specific research objective is essential. The summary of criteria will assist researchers to understand and apply IDA methods to different situations and data sources.

Shapley value is a fairly equitable solution to the cooperative game and passing symmetry, carrier and additivity axioms. Albrecht et al. (2002) first suggest applying Shapley value in IDA studies. However, the authors only introduce the concepts of Shapley value and do not formalize the characteristic functions. In Chapter 4, we extend the study of Albrecht et al. (2002) and formalize Shapley value in Laspeyres-based IDA methods to provide Shapley decomposition with desirable properties in energy studies.

Chapter 5 introduces some new findings on the properties and linkages of Divisia-based IDA methods. Ang (2004) points out a simple relationship between the additive and the multiplicative forms for both LMDI I and LMDI II. We further show that there exists a simple and meaningful relationship among most of the methods linked to the additive Divisia index, including that between AMDI and LMDI I. With these findings, we are able to extend the findings in Ang (2004). This improves our understanding of the properties of popular IDA methods and IDA methodology in general. The findings are also useful to analysts in method selection and decomposition result interpretation. In addition, desirable properties of the LMDI I method are proven, and LMDI I method is recommend as the preferred Divisia-based IDA methods.

Chapter 6 studies chaining and non-chaining approaches. Chaining and non-chaining approaches have been applied almost equally in IDA studies in recent years. In addition, there is no unified choice between chaining and non-chaining approaches in international organizations. For example, the International Energy Agency (IEA) updates energy efficiency studies using 1990 as the base year to track energy efficiency improvements, and a non-chaining approach is used, whereas the Office of Energy Efficiency (OEE) in Canada uses chaining approach to monitor energy efficiency improvement. Since application of these two approaches leads to different decomposition results, practitioners need a better understanding of the underlying issues and the implications of the choices they make. This study addresses some of these issues and provides recommendations.

Following the main studies, Chapter 7 contains the discussions and conclusions sections of this thesis as well as suggestions for future research.

CHAPTER 2: Literature Review of Index Decomposition Analysis

2.1 Introduction

As mentioned in Chapter 1, IDA is a technique used to study the impact of changes in a number of pre-defined factors of interest in energy-related aggregation. It was first applied to analyze the impact of industrial production mix shift on industrial energy demand by decomposing changes in the aggregate energy intensity into structural effect and intensity effect after the 1973/74 world oil crisis. It was found that structural effects can have a major impact on the aggregate energy intensity and that sectoral energy intensity was a better measure of energy efficiency than the aggregate energy intensity given by the total industrial energy demand to total industrial output. Since then, the application of IDA has been extended from only the industrial sector to economies as a whole, and from energy demand analysis to environmental analysis with more and more studies reported each year. The 1989 survey by Huntington listed only 11 studies including four journal papers, six conference papers and one PhD dissertation. Ang (1995a) surveyed 51 studies involving industrial energy decomposition analysis, and Ang and Zhang (2000) presented a comprehensive survey of IDA research which included 124 studies, with 109 studies related to IDA and 15 studies related to structure decomposition analysis (SDA). Some reported literature surveys confine studies to specific focuses. For instance, Ang (1999) reviewed 15 empirical studies (12 index decomposition and three structure decomposition) related to carbon emissions at the national and sectoral levels. Ma et al. (2010) reviewed 36 empirical studies related to energy intensity change in China.

So far, Ang and Zhang (2000) has been the most comprehensive survey on IDA. It covers a wide spectrum of IDA studies, both on the methodological and application fronts for energy and environmental analysis, providing a useful guide to researchers and practitioners. In the last ten years, more decomposition methods have been developed. For example, popular multiplicative LMDI I was proposed by Ang and Liu (2001). Additionally, some patterns of development in IDA studies have changed. For instance, the number of IDA studies dealing with CO_2 emissions has exceeded the number of IDA studies in energy demand, as a result of the growing emphasis on environmental protection and sustainable development worldwide. In addition, IDA studies have expanded substantially, with at least 170 new journal papers since Ang and Zhang (2000). Arising from these developments of IDA in both methodological and application aspects, it is significant to revisit the area and provide an up-to-date literature survey for future researchers as well as policy makers.

In this chapter, we first provide a historical overview of development of IDA in Section 2.2. We introduce formulae of IDA methods in Section 2.3. In Section 2.4, we refine the classification of IDA studies in Ang and Zhang (2000) and classify a total of 280 publications from 1978 to 2011 by application area, indicator type, decomposition method and several other attributes. A summary of new findings and observed main features is provided in Section 2.5.

2.2 Historical Overview of IDA

In general, we can divide the development of IDA in methodology into three phases: the beginning phase (prior to 1986), the development phase (1987-2001) and the refinement phase (from 2002 to now). In the beginning phase, researchers quantified the impact of structural shift in industrial energy demand intuitively and straightforwardly. Most of the popular IDA methods and tests for identifying desirable properties of IDA methods are proposed in the development phase. In the refinement phase, few new IDA methods are reported. Most of research work in methodology has been to refine and to consolidate IDA in theory.

2.2.1 The Beginning Phase

IDA studies used by researchers in the beginning phase (prior to 1986) are confined only to industrial energy demand and the methods applied are straightforward and intuitive. The impact of structural change was derived from the difference between the aggregate energy intensity in the target year, with sectoral energy intensities for all industrial sectors remaining at their base year and the aggregate energy intensity in the base year. The impact of energy intensity was singled out by the difference between the total change of aggregate energy intensity and the impact of structural shift. One early example of studies using this approach is Bossanyi (1979). The basic idea of calculating structural effect is to isolate the impact of structural shift from the change of aggregate energy intensity by changing this factor while holding all the other factors unchanged. Methodologically, it is similar to the Laspeyres index in economics. Therefore, it is referred as the Laspeyres IDA method.

2.2.2 The Development Phase

Most of the popular IDA methods are developed in this phase. Reitler et al. (1987) revised the Laspeyres IDA method by using the average of the base year and target year as the weight in the decomposition formulae, in contrast to assigning all the weight to the base year in the earlier studies. This new method enhances symmetry in the IDA formulae. Boyd et al. (1987) first introduced the Divisia index approach to IDA, and Boyd et al. (1988) proposed the AMDI method with discussions about the similarities of the classic economic index numbers and IDA. Since then, some popular IDA methods and tests have been derived from INP in economics.

In the early 1990s, various IDA methods had been developed. In an attempt to consolidate IDA methods into a unified decomposition framework, Liu et al. (1992a) propose two general parametric methods based on the Divisia index in additive decomposition and show that several of the methods proposed earlier, including Laspeyres/Paasche in Hankinson and Rhys (1983), the M-E method in Reitler et al. (1987) and the AMDI method in Boyd et al. (1988), are special cases of their two general parametric methods. A new method referred to as the Adaptive Weighting Divisia (AWD) method is also introduced to estimate the parameter values uniquely. Ang (1994) extends the work of Liu et al. (1992) to multiplicative decomposition and build a framework based on the two general parametric Divisia index methods for both additive and multiplicative decomposition approaches.

In the development of IDA, two common problems associated with the application of IDA have been discussed. The first issue is the interpretation of the residual term problem mentioned in Ang (1995a). Residual weakens the explanatory power of IDA, as this means a large part of the observed change in the aggregate energy indicator being decomposed is left unexplained. The residential problem tends to be more serious in those IDA studies using Laspeyres-based methods. Zero value in the data set may also lead to computational problems in some Divisia-based IDA methods. To solve these two problems, perfect IDA methods with no residual and research work on zero value were studied. Ang and Choi (1997) propose a refined Divisia method (LMDI II) based on the multiplicative form. Ang et al. (1998) introduce the additive LMDI I method and the multiplicative version is studied in Ang and Liu (2001). Sun (1998) proposes a Laspeyres-based IDA method with the residual distributed equally among the main effects based on the "jointly created and equally distributed" principle. LMDI I and II and Sun's method are all perfect and leave no residuals. Ang and Choi (1997) and Ang et al. (1998) discuss the zero value problem and show that this problem could be overcome by replacing zero values with a small positive number, since converging results are generally obtained as the small positive number approaches zero.

2.2.3 The Refinement Phase

Most of the popular IDA methods are proposed prior 2002. In the refinement phase, research work in methodology is mainly to refine and to consolidate IDA in theory.

Albrecht et al. (2002) link IDA with the Shapely value in game theory and propose a perfect method. Ang et al. (2003) prove that the Shapley decomposition technique and the method by Sun (1998) are exactly the same mathematically. Therefore, this method is named the Shapley/Sun (S/S) method. Liu and Ang (2003) study the similarities of IDA and INP and introduce the conventional two-factor Fisher index to IDA. Ang et al. (2004) develop a generalized Fisher index method from Shapley value aspect to extend the conventional two-factor Fisher index decomposition approach to more than two factors. Ang (2004) classifies IDA methods into two groups, i.e. one based on the concept of the Divisia index, and the other based on that of the Laspeyres index or index numbers linked to the Laspeyres index. This paper also points out a simple relationship between the additive and the multiplicative forms for both LMDI I and LMDI II. Ang and Liu (2007b) and Ang and Liu (2007c) describe a set of guidelines to deal with all possible cases of changes that involve negative and/or zero values, using the analytical limit strategy for the LMDI approach and consolidate special value problem in IDA. Ang et al. (2009) study the properties and linkages of some popular IDA methods in energy and carbon emission analysis. It is found that most Divisia-based IDA methods collapse to LMDI I in additive after applying the "proportionately distributed by sub-category" to the residual terms.

2.3 Formulae of IDA Methods

Following Ang (2005), let V be an energy-related aggregate and we wish to study the underlying factors contributing to its changes over time. Assume that there are n factors contributing to the changes in V and each is given by a quantifiable variable whereby n variables, $x_1, x_2, x_3, \ldots, x_n$, are specified. Let subscript j be a sub-category of the aggregate for which changes related to a certain structure, such as activity mix or fuel mix, are to be studied among other effects. It is also assumed that at the sub-category level the relationship $V_j = x_{j,1} \cdot x_{j,2} \cdot x_{j,3} \cdot \ldots \cdot x_{j,n}$ holds.

The general IDA identity is then given by

$$V = \sum_j V_j = \sum_j x_{j,1} \cdot x_{j,2} \cdot x_{j,3} \cdot \ldots \cdot x_{j,n} \tag{2-1}$$

Further assume that the aggregate energy-related changes from $V^0 = \sum_j V_j^0 = \sum_j x_{j,1}^0 \cdot x_{j,2}^0 \cdot x_{j,3}^0 \cdot \ldots \cdot x_{j,\mathrm{n}}^0$ in time period 0 to $V^T = \sum_j V_j^T = \sum_j x_{j,1}^T \cdot x_{j,2}^T \cdot x_{j,3}^T \cdot \ldots \cdot x_{j,\mathrm{n}}^T$ in period T.

2.3.1 Additive IDA Methods

In additive decomposition, we decompose the difference of an energy-related aggregate V from time 0 to time T as:

$$\Delta V_{tot}^{0,T} = V^T - V^0 = \sum_{i=1}^{n} \Delta V_{x_i}^{0,T} + \Delta V_{rsd}^{0,T} \tag{2-2}$$

where, the subscript *tot* denotes the total or overall difference change, *rsd* denotes the residual and the terms on the right-hand side give the effects associated with the respective factors in Eq.(2-1). $\Delta V_{x_1}^{0,T}$, $\Delta V_{x_2}^{0,T}$, …, $\Delta V_{x_n}^{0,T}$ are the estimated impacts of factor 1,2,…,*n*, respectively. Normally, the sum of all the estimated effects will not be equal to the total difference change; therefore, there is a residue term $\Delta V_{x_{rsd}}^{0,T}$. When the IDA method is perfect, $\Delta V_{x_{rsd}}^{0,T}$ is equal to 0.

The relative change from consecutive years *t* to *t*+1, where *t* is an integer belonging to [0, *T*-1], is given by:

$$\Delta V_{tot}^{t,t+1} = V^{t+1} - V^{t} = \sum_{i=1}^{n} \Delta V_{x_i}^{t,t+1} + V_{rsd}^{t,t+1} \tag{2-3}$$

Therefore, in additive decomposition, when using the non-chaining approach, the decomposition effect of a factor is obtained directly using decomposition methods in Eq. (2-2); whereas using the chaining approach, the decomposition effect of factor is computed on a cumulative basis over time. Take factor *i* as an example

$$\Delta V_{x_i}^{0,T} = \sum_{t=0}^{T-1} \Delta V_{x_i}^{t,t+1} \tag{2-4}$$

2.3.2 Multiplicative IDA Methods

In multiplicative decomposition, we decompose the ratio change of an energy-related aggregate *V* from time 0 to time *T* as:

$$D_{tot}^{0,T} = V^T / V^0 = \prod_i D_{x_i}^{0,T} D_{rsd}^{0,T} \tag{2-5}$$

where, the subscript *tot* denotes the total or overall ratio change, *rsd* denotes the residual and the terms on the right-hand side give the effects associated with the respective factors in Eq.(2-1). $D_{x_1}^{0,T}$, $D_{x_2}^{0,T}$, …, $D_{x_n}^{0,T}$ are the estimated impacts of factor 1,2,…,*n* respectively. Normally, the product of all the estimated effects will not be equal to the total ratio change; therefore, there is a residue term $D_{rsd}^{0,T}$. When the IDA method is perfect, $D_{rsd}^{0,T}$ is equal to 1.

The relative change from consecutive years *t* to *t*+1, where *t* is an integer belonging to [0, *T*-1], is given by:

$$D_{tot}^{t,t+1} = V^{t+1} / V^t = \prod_i D_{x_i}^{t,t+1} D_{rsd}^{t,t+1} \tag{2-6}$$

Therefore, in multiplicative decomposition, when using the non-chaining approach, the decomposition effect of a factor, is obtained directly using decomposition methods in Eq. (2-5); whereas using the chaining approach, the decomposition effect of factor is computed on a cumulative basis over time. Take factor *i* for example

$$D_{x_i}^{0,T} = \prod_{t=0}^{T-1} D_{x_i}^{t,t+1} \tag{2-7}$$

2.3.3 Laspeyres-based IDA Methods

As shown in Figure 1-1, the main Laspeyres-based methods include Laspeyres, Paasche, and M-E. The perfect additive Laspeyres-based method is the S/S method and the perfect multiplicative Laspeyres-based method is Fisher Ideal for the 2-factor cases and is generalized Fisher for n-factor cases. The detailed study of Laspeyres-based methods is provided in Chapter 4.

The popular Laspeyres-based IDA methods are Laspeyres and S/S. We classify the Laspeyres-based IDA methods into three categories: Laspeyres, S/S and other Laspeyres-based IDA methods. We will describe the formulae of Laspeyres and S/S methods below respectively.

Laspeyres Method

Laspeyres is the very first method used in IDA (Myers and Nakamura, 1978). The basic idea is to isolate the impact of a certain variable to the change of an energy-related aggregate indicator by changing the impacting variable while holding other variables unchanged.

Based on the general IDA case for n factors and m sub-categories described above, the formulae of additive Laspeyres method for factor x_i and residual term between year 0 and year T are:

$$\Delta V_{x_i}^{0,T} = \sum_{j=1}^{m} \frac{V_j^0}{x_{j,i}^0} \cdot (x_{j,i}^T - x_{j,i}^0) \tag{2-8}$$

$$\Delta V_{rsd}^{0,T} = \Delta V_{tot}^{0,T} - \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{V_j^0}{x_{j,i}^0} \cdot (x_{j,i}^T - x_{j,i}^0) \tag{2-9}$$

The formulae of multiplicative Laspeyres method for factor x_i and residual term between year 0 and year T are:

$$D_{x_i}^{0,T} = \frac{\sum_{j=1}^{m} \frac{V_{j,i}^{0}}{x_{j,i}^{0}} \cdot x_{j,i}^{T}}{\sum_{j=1}^{m} V_j^0} \tag{2-10}$$

$$D_{rsd}^{0,T} = D_{tot}^{0,T} \Big/ \prod_{i=1}^{n} D_{x_i}^{0,T} \tag{2-11}$$

S/S Method

Sun (1998) introduces the principle of "jointly created and equally distributed" to distribute the combinatorial residual terms (calculated from Laspeyres method) among the main effects. Albrecht et al. (2002) apply the Shapely value technique in IDA studies. Ang et al. (2003) prove that these two methods have the exactly same results.

We present the S/S method using the formula given in Sun (1998) in this Section and the formula of S/S method given in Albrecht et al. (2002) will be provided in Chapter 4. Based on the general IDA case for n factors and m sub-categories described above, the formula of S/S method for factor x_i between year 0 and year T is:

$$\Delta V_{x_i}^{0,T} = \sum_{j=1}^{m} \sum_{r=1}^{n} \sum_{\substack{l_1, l_2, \cdots, l_r \\ i = l_1}} \frac{V^0}{x_{j,l_1}^0 x_{j,l_2}^0 \cdots x_{j,l_r}^0} \Delta x_{j,l_1} \Delta x_{j,l_2} \cdots \Delta x_{j,l_r} / r \tag{2-12}$$

Where $l_1 \neq l_2 \neq \cdots \neq l_r$, $l_k \in [1, n]$, $\Delta x_{j,l_k} = x_{j,l_k}^T - x_{j,l_k}^0$, for $k = 1, 2, \cdots, r$.

2.3.4 Divisia-based IDA Methods

As shown in Figure 1-1, the main Divisia-based methods include LMDI I, LMDI II, AMDI, LAS-PDM1, PAA-PDM1 and AVE-PDM1. Among these methods, LMDI I and LMDI II are perfect in decomposition while the other four methods are not. The detail study of Divisia-based IDA methods is provided in Chapter 5.

The popular Divisia-based IDA methods are LMDI I, LMDI II and AMDI. We classify Divisia-based IDA methods into three categories: LMDI (LMDI I and LMDI II), AMDI and other Divisia-based IDA methods. We will describe the formulae of LMDI I, LMDI II and AMDI below respectively.

LMDI I Method

Based on the general IDA case for n factors and m sub-categories described above, the formula of additive LMDI I method for factor x_i between year 0 and year T is:

$$\Delta V_{x_i}^{0,T} = \sum_{j=1}^{m} L(V_j^T, V_j^0) \cdot \ln \frac{x_{j,i}^T}{x_{j,i}^0} \tag{2-13}$$

The formula of multiplicative LMDI I method for factor x_i between year 0 and year T is:

$$D_{x_i}^{0,T} = \exp\left\{ \sum_{j=1}^{m} L\left(\frac{V_j^T}{V^T}, \frac{V_j^0}{V^0}\right) \cdot \ln \frac{x_{j,i}^T}{x_{j,i}^0} \right\} \tag{2-14}$$

The logarithmic average of two positive numbers a and b is defined by

$$L(a,b)=\begin{cases}\dfrac{a-b}{\ln a-\ln b}, & \text{for } a\neq b\\ a, & \text{for } a=b\end{cases} \tag{2-15}$$

LMDI II Method

Based on the general IDA case for n factors and m sub-categories described above, the formula of additive LMDI II method for factor x_i between year 0 and year T is:

$$\Delta V_{x_i}^{0,T}=\sum_{j=1}^{m}\frac{L(\frac{V_j^T}{V^T},\frac{V_j^0}{V^0})}{\sum_j L(\frac{V_j^T}{V^T},\frac{V_j^0}{V^0})}\cdot L(V^0,V^T)\cdot\ln\frac{x_{j,i}^T}{x_{j,i}^0} \tag{2-16}$$

The formula of multiplicative LMDI II method for factor x_i between year 0 and year T is:

$$D_{x_i}^{0,T}=\exp\left\{\sum_{j=1}^{m}\frac{L(\frac{V_j^T}{V^T},\frac{V_j^0}{V^0})}{\sum_j L(\frac{V_j^T}{V^T},\frac{V_j^0}{V^0})}\cdot\ln\frac{x_{j,i}^T}{x_{j,i}^0}\right\} \tag{2-17}$$

AMDI Method

Based on the general IDA case for n factors and m sub-categories described above, the formulae of additive AMDI method for factor x_i and residual term between year 0 and year T are:

$$\Delta V_{x_i}^{0,T} = \sum_{j=1}^{m} \frac{1}{2}(V_j^T + V_j^0) \cdot \ln \frac{x_{j,i}^T}{x_{j,i}^0} \tag{2-18}$$

$$\Delta V_{rsd}^{0,T} = \Delta V_{tot}^{0,T} - \sum_{i=1}^{n} \Delta V_{x_i}^{0,T} = \Delta V_{tot}^{0,T} - \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{1}{2}(V_j^T + V_j^0) \cdot \ln \frac{x_{j,i}^T}{x_{j,i}^0} \tag{2-19}$$

The formulae of multiplicative AMDI method for factor x_i and residual term between year 0 and year T are:

$$D_{x_i}^{0,T} = \exp\left\{ \sum_{j=1}^{m} \frac{1}{2} \cdot \left(\frac{V_j^T}{V^T} + \frac{V_j^0}{V^0}\right) \cdot \ln \frac{x_{j,i}^T}{x_{j,i}^0} \right\} \tag{2-20}$$

$$D_{rsd}^{0,T} = D_{tot}^{0,T} \Big/ \prod_{i=1}^{n} \exp\left\{ \sum_{j=1}^{m} \frac{1}{2} \cdot \left(\frac{V_j^T}{V^T} + \frac{V_j^0}{V^0}\right) \cdot \ln \frac{x_{j,i}^T}{x_{j,i}^0} \right\} \tag{2-21}$$

2.4 Main Features of Past Studies

In Ang and Zhang (2000), journal papers, important conference papers and book chapters related to IDA are all included. Due to the extensive development of IDA, recent studies related to IDA have been widely reported in various journals, books, conference papers and national reports. To capture the key information about the development of IDA and to maintain a sound quality of the selected studies, we confine the scope of IDA studies to journal papers and important books. Conference papers and reports are excluded from this survey. In this Chapter, a total of 280 IDA studies have been reviewed and summarized in Table 2-4.

IDA studies are listed by year of publication to show the chronological development of IDA research. This review provides information of country/region under study, application area, indicator type, number of sector, decomposition approach and decomposition method. In terms of “application area”, IDA studies are broadly divided into three categories: energy demand analysis, GHG emission analysis and other application areas. For each category, it is further divided into economy-wide studies, industry studies and studies such as transportation and electricity generation sectors exclusive of industry. For the decomposition scheme, IDA studies are reviewed by decomposition approach (additive IDA studies, multiplicative IDA studies) and decomposition method (Laspeyres-based IDA methods, Divisia-based IDA methods, or other IDA methods).

Since the first IDA study in 1978, IDA has been developed for more than thirty years. We classified the studies into seven time periods that are roughly equal in time span (1978-1981, 1982-1986, 1987-1991, 1992-1996, 1997-2001, 2002-2006, and 2007-2011) to show the possible changes over time. The numbers of studies in these periods are, respectively, 2, 9, 14, 29, 61, 65, 100, which show significant increases over time, particularly in the last sub-periods.

2.4.1 Application Area

Energy demand and energy-related gas emission are two major application area of IDA. IDA studies have been applied for various energy types, such as primary energy consumption, final energy consumption, electricity and the consumption of individual fuel types. Most of energy-related gas emission IDA studies are about CO_2 emission. Other types of GHG emissions (SO2, NOx or others) are also studied. Examples are He (2010) and Viguier (1999). With the development of IDA in energy and energy-related gas emissions, the application has been extended to other area, which is listed in the column "others" under the main column "application area". Some examples are given as follows. Lai et al. (1998) describe the use of a decomposition technique to single out the product-mix effect and the effect associated with changes in real process performance for the aggregate fraction defective in batch/short run production. Hoffrén et al. (2000) use decomposition analysis to evaluate the trend in material use and provide a basis for assessments of sustainability for Finland. Luyanga et al. (2006) apply IDA to study changes in Namibian aggregate water intensity between 1993 and 2001.

In Ang and Zhang (2000), the application areas are sub further divided into two groups: industry and other application areas. The latter includes economy-wide IDA studies and studies focusing on specific sectors such as transportation and electricity generation. Tracking and monitoring economy-wide energy efficiency is popular in recent years. Therefore, we update the classification of application area of Ang and Zhang (2000) and classify application areas into three groups: economy-wide, industry and other economy sectors exclusive of industry.

The last row of Table 2-4 gives the information of total counts of IDA studies for each column. Some IDA studies deal with more than one application areas. For example Ang (2005) studies both industrial energy consumption and CO_2 emission. Kaivo-oja and Luukkanen (2004) studies both economy-wide energy demand and CO_2 emissions. Therefore, the total count of application area is larger than the total number of reviewed IDA studies.

Figure 2-1 and Figure 2-2 show the changes that have taken place by application area. From Figure 2-1, we can see that there has been a substantial increase in the number of gas emission studies since it was first studied in Torvanger (1991). In Ang and Zhang (2000), although the share taken up by emission studies increased substantially, gas emission was not the dominant application area. However, in this new survey, from Figure 2-2, we can find that gas emission has been the main application area since 2000. It is consistent with the growing concern on environmental protection and sustainable development worldwide.

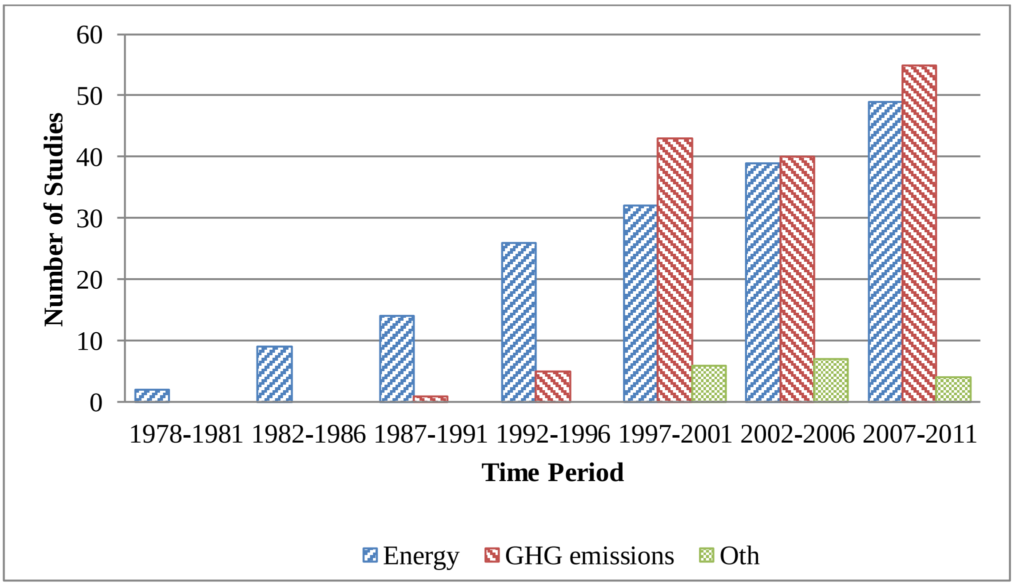

Figure 2-1. Number of studies by application area over time

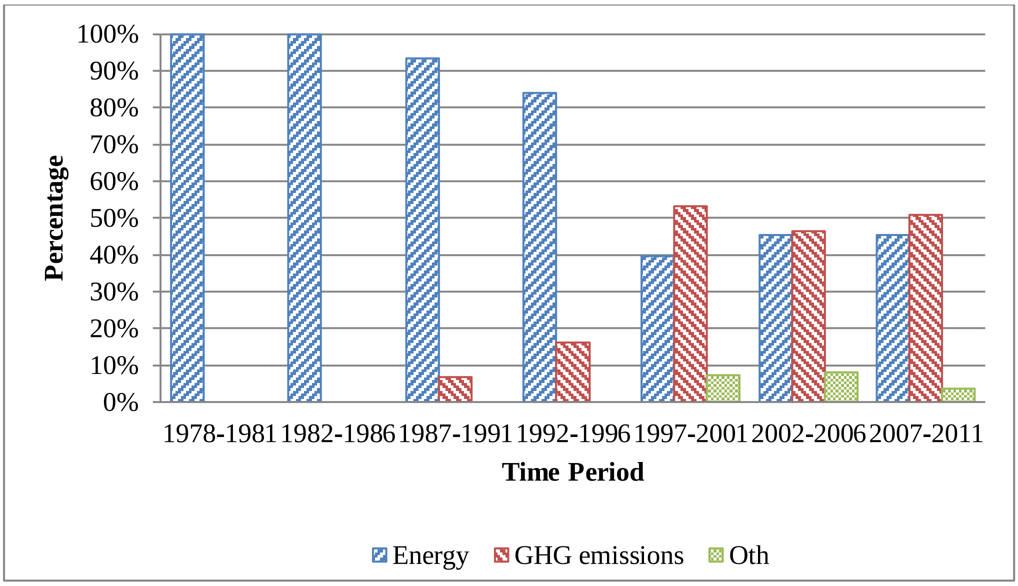

Figure 2-2. Share of studies by application area over time

Figure 2-3 and Figure 2-4 show the number and the share of publications by sector over time respectively, where "Eco" refers to economy-wide studies, "Ind" refers to industry sector studies and "Oth" denotes other studies. From Figure 2-3, it is found that there is a substantial increase in the number of each application sector from 1978 to 2011. Although in absolute terms, the number of publications in industrial sector remains increasing, the share taken up by industrial sector drops from 100% to 38%, while, the share taken up by economy-wide increase from 0% to 44%. This shift shows that the main application sector switches from industry to economy-wide. The extension of application area of IDA studies is clearly the result of the wide acceptance of IDA in tracking economy-wide energy efficiency trends with its development in theory.

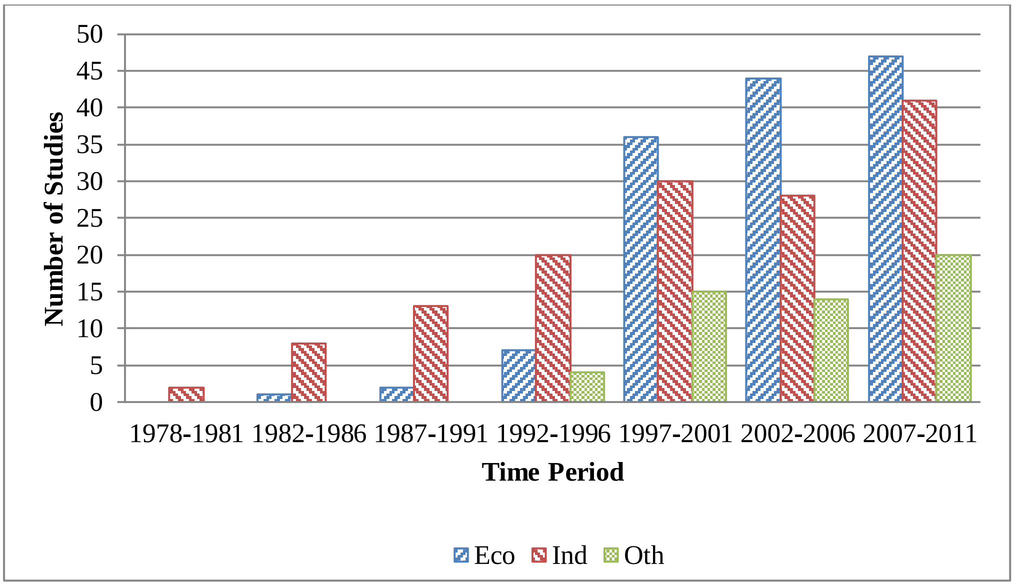

Figure 2-3. Number of studies by sector over time

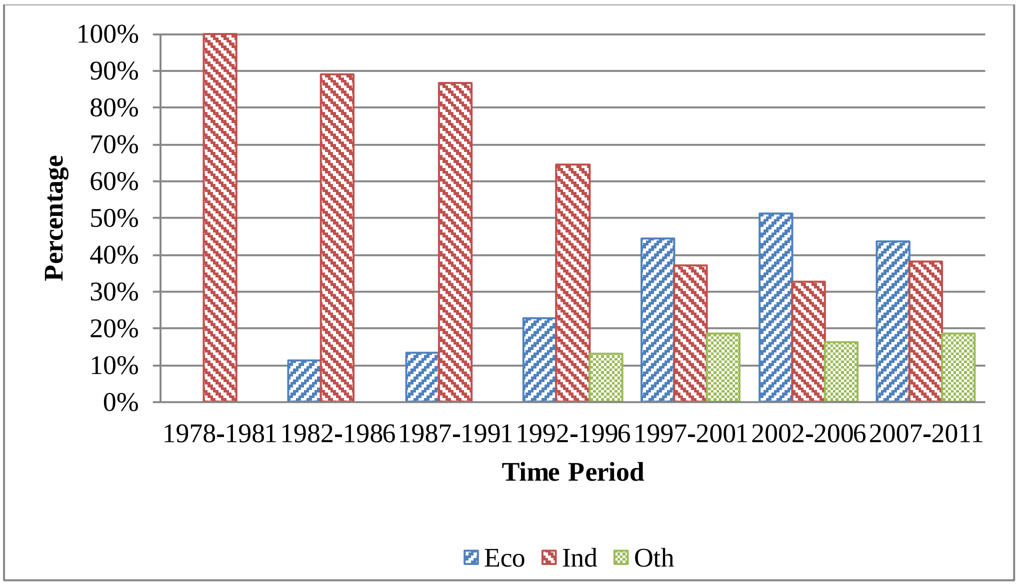

Figure 2-4. Share of studies by sector over time

2.4.2 Indicator Type

In this review, indicator type is classified into three categories: quantity indicator, ratio indicator and other indicators. Quantity indicator refers to aggregates such as total energy consumption and total gas emissions, where a single physical measurement unit is involved. Quantity indicators are direct descriptions of energy-related matters and have the advantage of enabling the ease of understanding. Ratio or index indicator includes aggregate energy intensity and aggregate gas emission intensity which are expressed in indices. The advantage of ratio indicators is that they deal with relations between the quantity indicators and these relations may provide additional information that cannot be revealed by quantity indicators. There are other indicators, which are seldom used in decomposition studies. One example is energy elasticity, or energy coefficient: the ratio between the growth rates of energy consumption and gross domestic production (GDP). These other indicators are included in the “Oth” group.

Figure 2-5 and Figure 2-6 show the number and share of indicator types over time respectively, where “Q” denotes quantity indicator and “R/I” denotes ratio or index indicator. We can see that the ratio and quantity indicators are widely applied measures and that other indicators are rarely used. From Figure 2-5, it is found that quantity indicator increases substantially over time periods while the ratio indicator remains fairly constant in the last three time periods. From Figure 2-6, it is found that at the beginning of IDA, main indicator was ratio indicator. However, quantity indicator becomes more widely used in recent years.

Ang and Zhang (2000) conclude that the number of studies using quantity indicator was about the same as that using ratio indicator. In this new survey, it is concluded that quantity indicator is becoming more popular. The reason may be that quantity indicators are easier to interpret compared to ratio indicators.

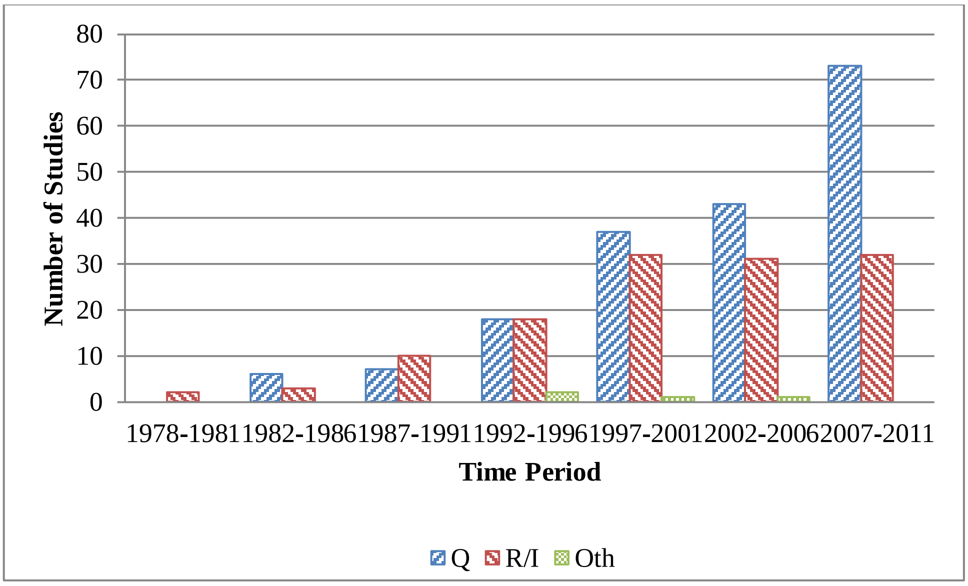

Figure 2-5. Number of studies by indicator type over time

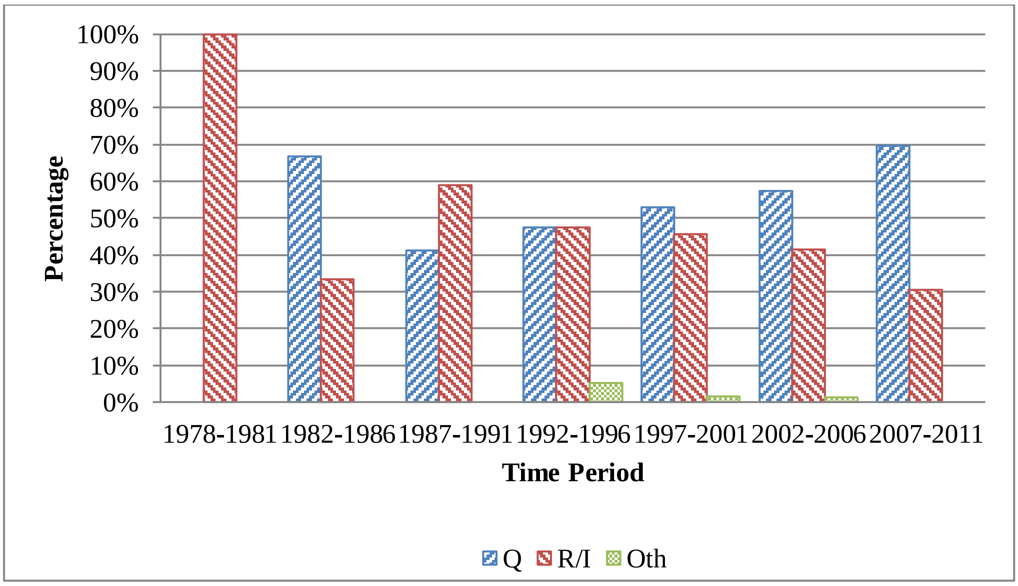

Figure 2-6. Share of studies by indicator type over time

2.4.3 Decomposition Approach

As introduced in Chapter 1, decomposition approach could be divided into two categories: additive and multiplicative decomposition approaches.

Figure 2-7 shows the number of publications by decomposition approach: multiplicative and additive approaches, where “Mul” denotes multiplicative decomposition and “Add” denotes additive decomposition. From this figure, it is found that there is a substantial increase in the number of each decomposition approach from 1978 to 2011.

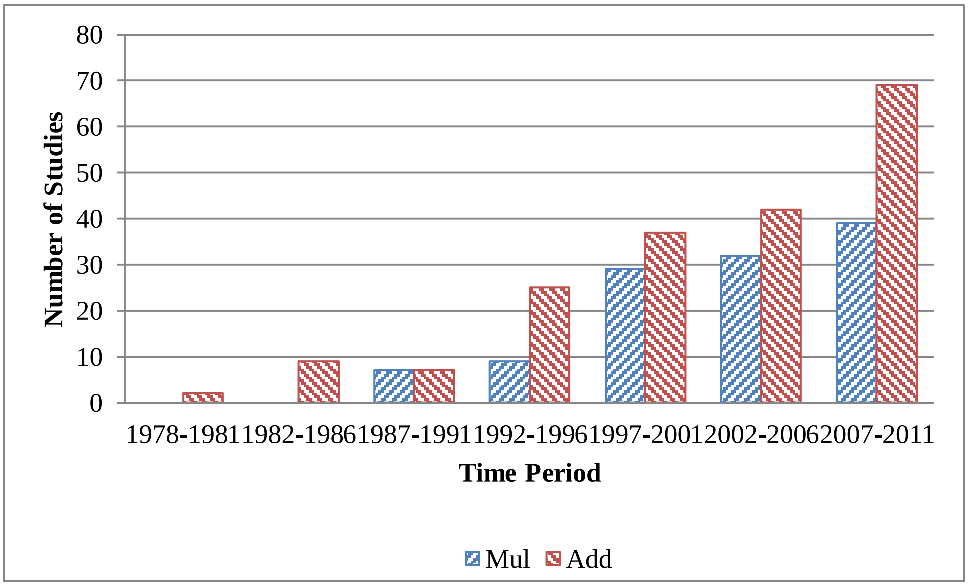

Figure 2-7. Number of studies by decomposition approach over time

Figure 2-8 shows the share of publications by decomposition approach over time. It is found that the additive approach is always the dominant one.

Ease of result interpretation may be the reason why additive approach is often preferred by analysts.

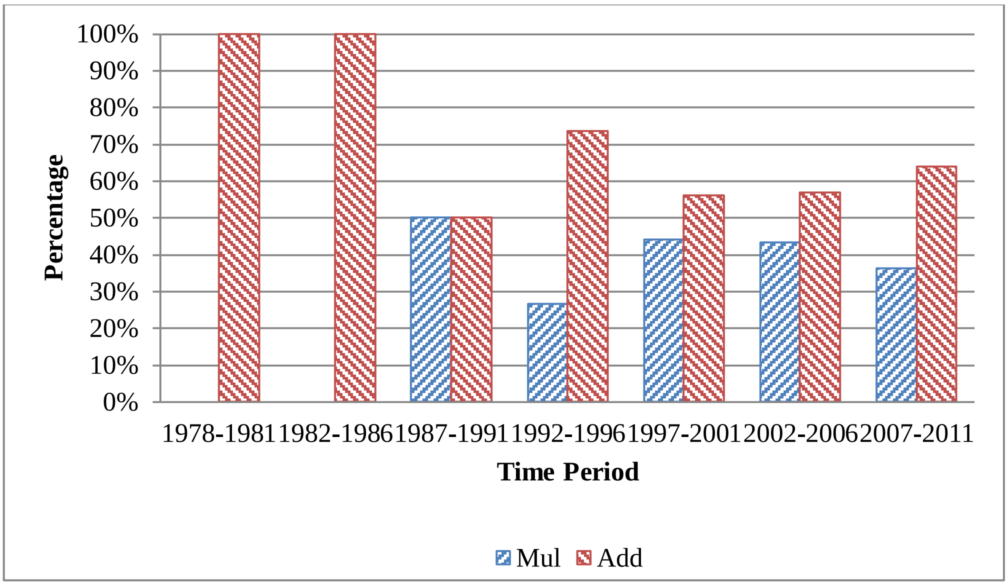

Figure 2-8. Share of studies by decomposition approach over time

2.4.4 Chaining and Non-chaining

Chaining and non-chaining are two different indexing approaches in IDA to treat time. In chaining approach, IDA results are computed in a cumulative manner from the decomposition results obtained based on the data of two consecutive years. In some IDA studies, the set consists of results using the data for years 0 and 1, 0 and 2, and so on till 0 and T. In this kind of situation, although time series results are given, we still classify the study as a non-chaining study. In some cases, period-wise results are given. However, the decomposition result of the whole time period is the accumulation of the decomposition results of several continuous time periods (not yearly results).

We regard these cases as using chaining approach. When a study has only two years' data, we regard the study as using non-chaining approach. In this review, 11 studies are excluded as they do not specify whether chaining or non-chaining methods was used.

Figure 2-9 and Figure 2-10 respectively show the number and the share of publications by treatment of time over time, where "Cha" denotes chaining approach and "Non-cha" denotes non-chaining approach. The numbers of studies of both non-chaining and chaining approaches increase substantially. The share of non-chaining approach declines slowly over time, while the share of chaining approach increases slowly over time. In the latest time span, chaining and non-chaining approaches have comparable popularity.

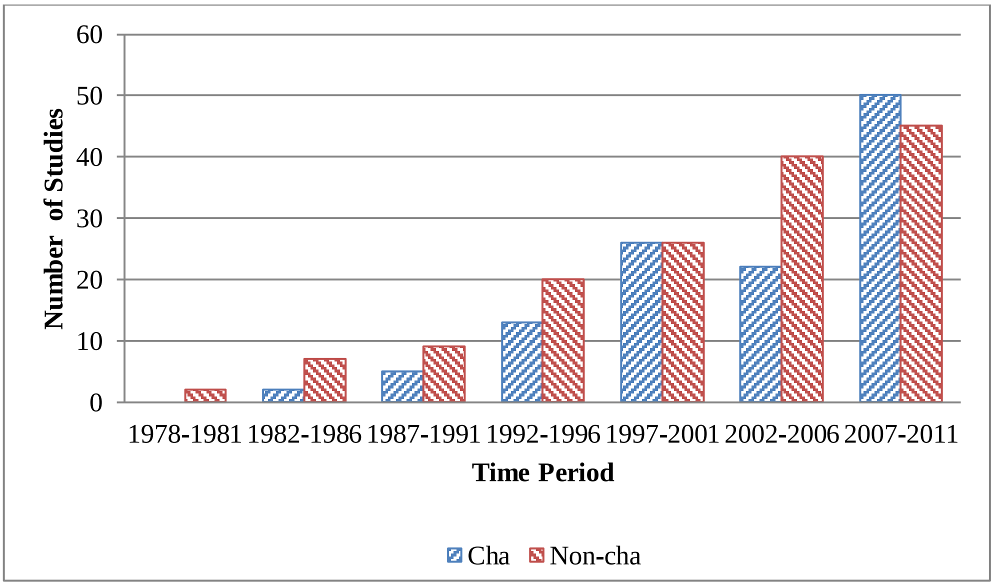

Figure 2-9. Number of studies by treatment of time over time

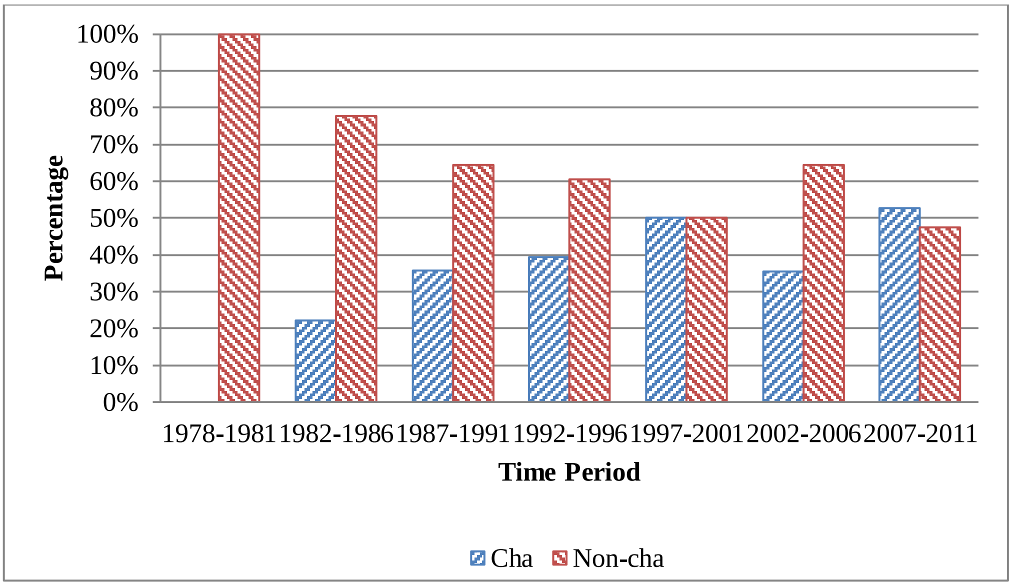

Figure 2-10. Share of studies by treatment of time over time

2.4.5 Decomposition Methods

As discussed in Chapter 1, the IDA methods are divided into three categories: Laspeyres-based IDA methods, Divisia-based IDA methods and other methods. Divisia-based IDA methods are further classified into LMDI, AMDI and other Divisia-based method. Laspeyres-based methods are further classified into Laspeyres, S/S and other Laspeyres-based methods. "Other methods" include the path based approach, an extension to Sun's decomposition (Fernandez and Fernandez, 2008), MRCI method (Lenzen, 2006), Stuvel index method (Liu and Ang, 2003) and so on.

Some past studies employed formulae of the general parametric Divisia methods, PDM1 and PDM2, for additive or multiplicative decomposition approaches (Liu et al. 1992a). Additive AVE-PDM1 is the same as the AMDI, additive LAS-PDM2 is the same as the Laspeyres method, additive PAA-

PDM2 is the same as the Paasche method, and additive AVE-PDM2 is the same as the M-E method. Other additive PDM1 and PDM2 methods listed in Ang (1994), such as LAS-PDM1, PAA-PDM1 and AWD, belong to "other Divisia methods". Multiplicative AVE-PDM1 is the same as the multiplicative AMDI method, and other multiplicative PDM1 and PDM 2 methods listed in Ang (1994) belong to "other Divisia methods".

Figure 2-11 and Figure 2-12 show the number and the share of studies using IDA methods over time respectively. From Figure 2-11, it is found that both the number of studies using the Divisia-based methods and the number of studies using the Laspeyres-based methods increase substantially over time. From Figure 2-12, it is found that the share of the Divisia-based IDA methods increases, while the share of the Laspeyres-based IDA decreases substantially over time. In the last ten years, the dominant methods change from the Laspeyres-based methods to the Divisia-based methods.

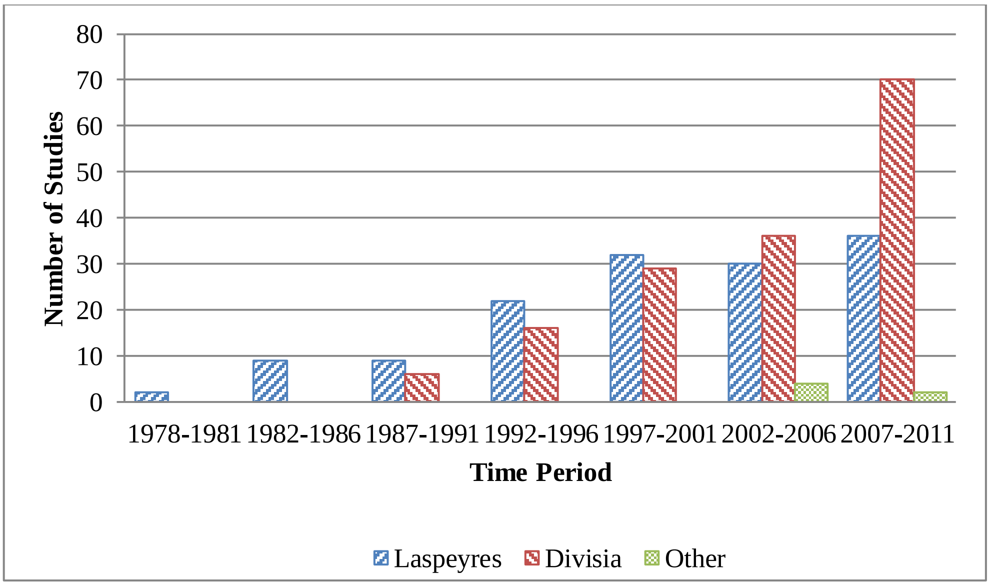

Figure 2-11. Number of studies by decomposition methods over time

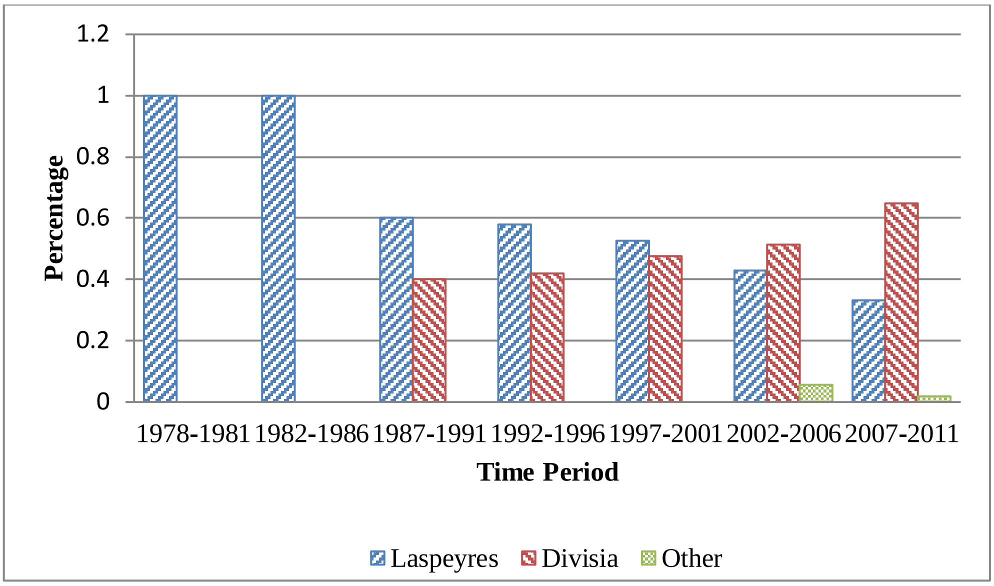

Figure 2-12. Share of studies by decomposition methods over time

Figure 2-13 and Figure 2-14 show the number and the share of studies using the Laspeyres-based methods respectively, where “Las” denotes Laspeyres method and “S/S” denotes the S/S method. The number of studies using the S/S method increases substantially after it was proposed. The share of the Laspeyres method decreases over time. In the last two time spans, S/S becomes the most popular Laspeyres-based method.

Figure 2-15 and Figure 2-16 show the number and the share of studies using the Divisia-based methods over time respectively. The number of studies using AMDI has remained almost unchanged during the last 20 years, while the number of studies using LMDI increases rapidly since it was first presented by Ang and Choi (1997). The share of AMDI method decreases while the share of LMDI increases substantially over time. In the last two time spans, the most popular Divisia-based method changes from AMDI to LMDI

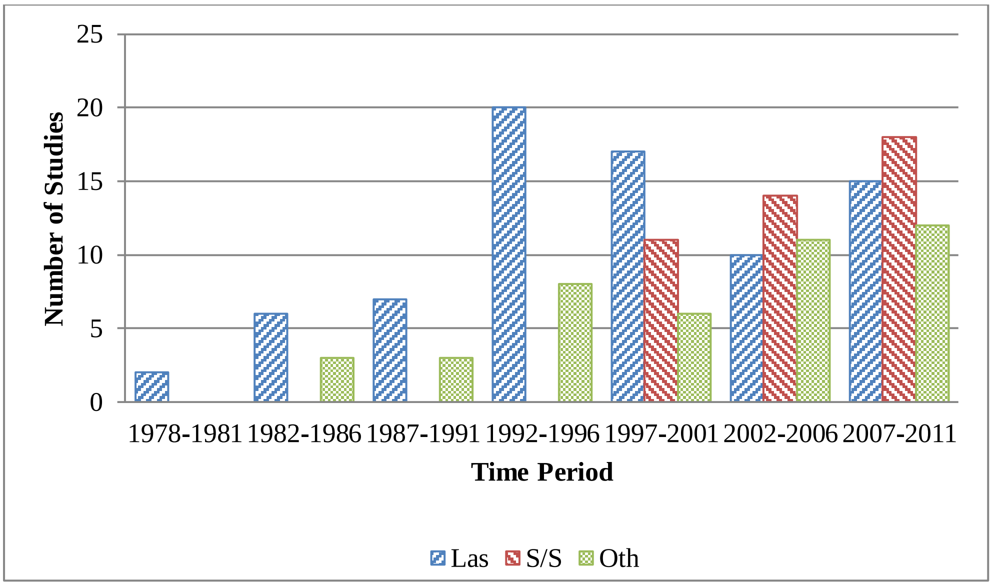

Figure 2-13. Number of studies using Laspeyres-based methods over time

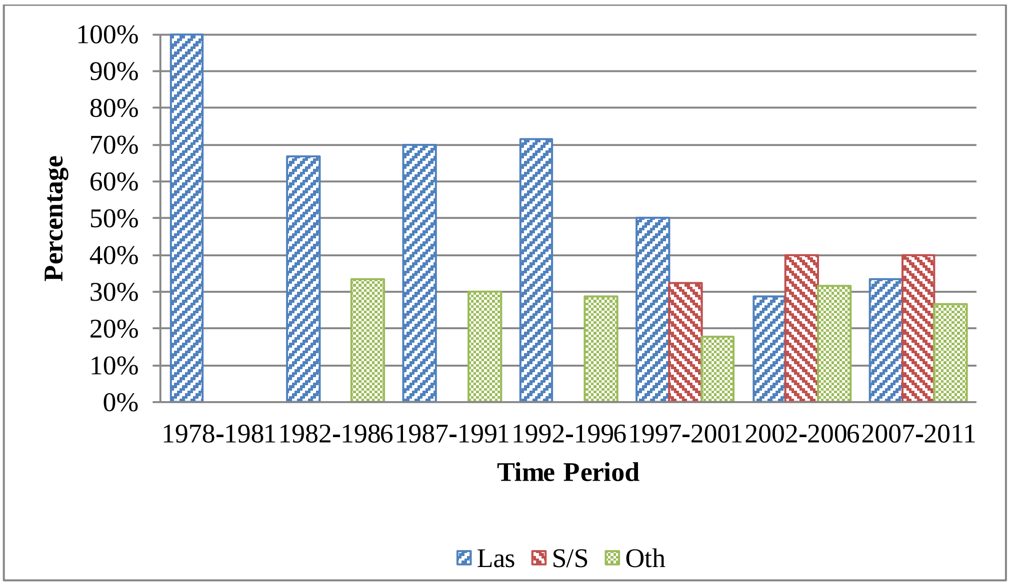

Figure 2-14. Share of studies using Laspeyres-based methods over time

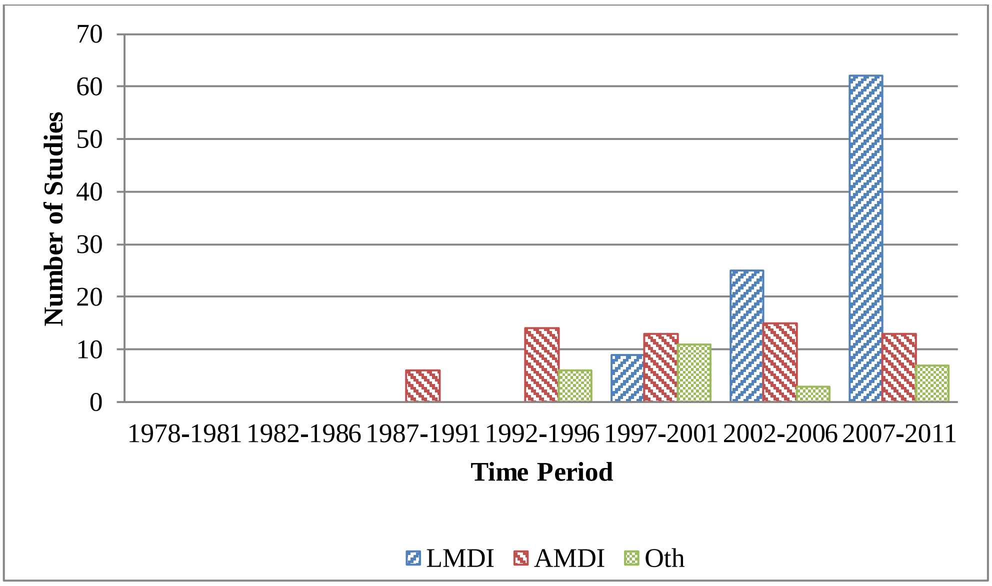

Figure 2-15. Number of studies using Divisia-based methods over time

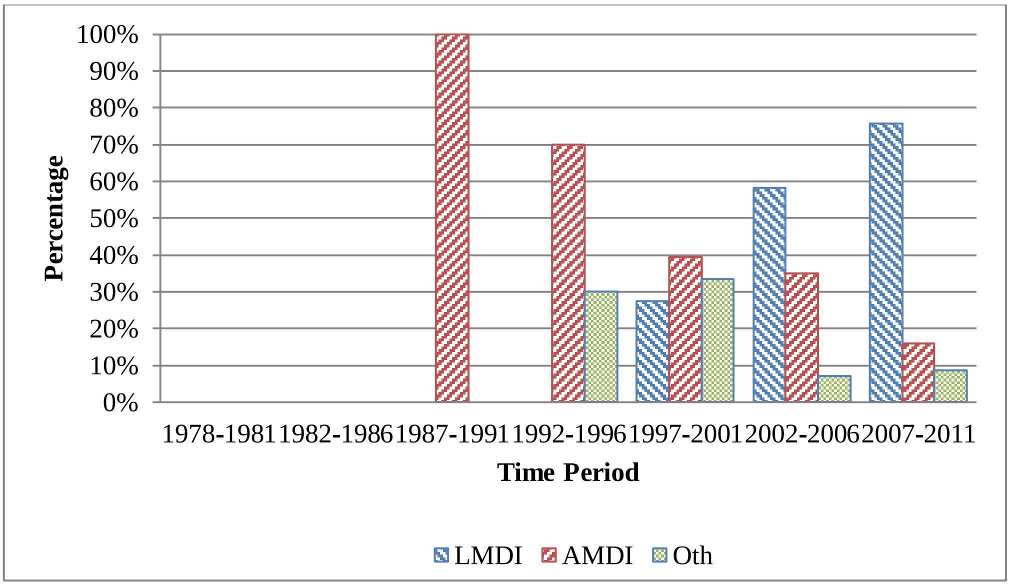

Figure 2-16. Share of studies using Divisia-based methods over time

2.4.6 Level of Disaggregation

Level of disaggregation shows the sectors into which an economy is disaggregated. In general, the finer the level of disaggregation, the better are the estimates of the factor effects in IDA. However, it is difficult to implement high disaggregation levels in practice, since level of disaggregation is dependent on data availability and quality. In IDA, when a study uses only a specific level of disaggregation, we refer to it as a single-level decomposition study; when a study uses more than one level of disaggregation, we refer to it as a multi-level study.

Figure 2-17 and Figure 2-18 show the number and the share of studies by level of disaggregation over time, where "S" refers to single-level studies and "M" refers to multi-level studies. From Figure 2-17, we can see that the number of studies using single-level increases substantially over time, and the number of studies using multi-level only increases slowly. From Figure 2-18, we find that the single-level is the dominant level of disaggregation. The level of disaggregation selection is often dependent on data availability and this may be the reason why the number of studies using multi-level only increases slowly and the single-level is the dominant level of disaggregation.

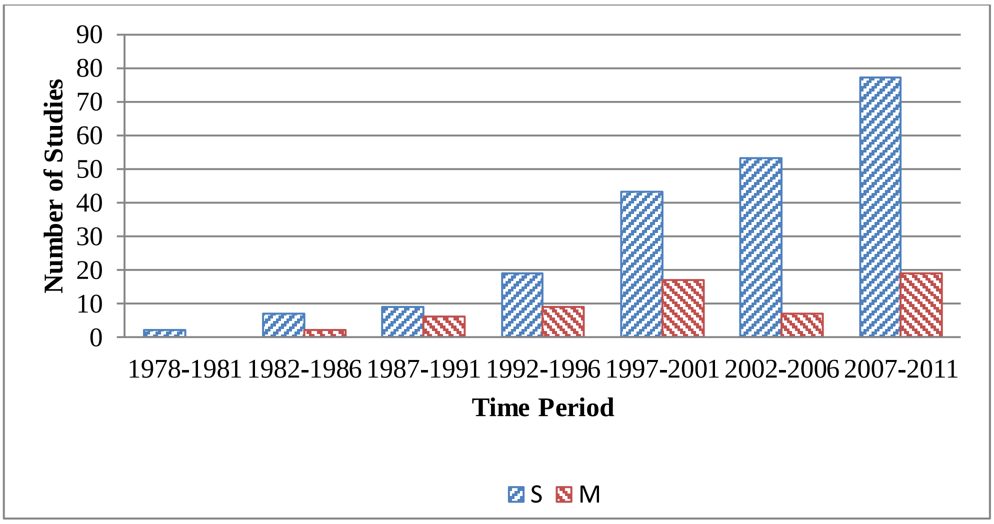

Figure 2-17. Number of studies by level of disaggregation over time

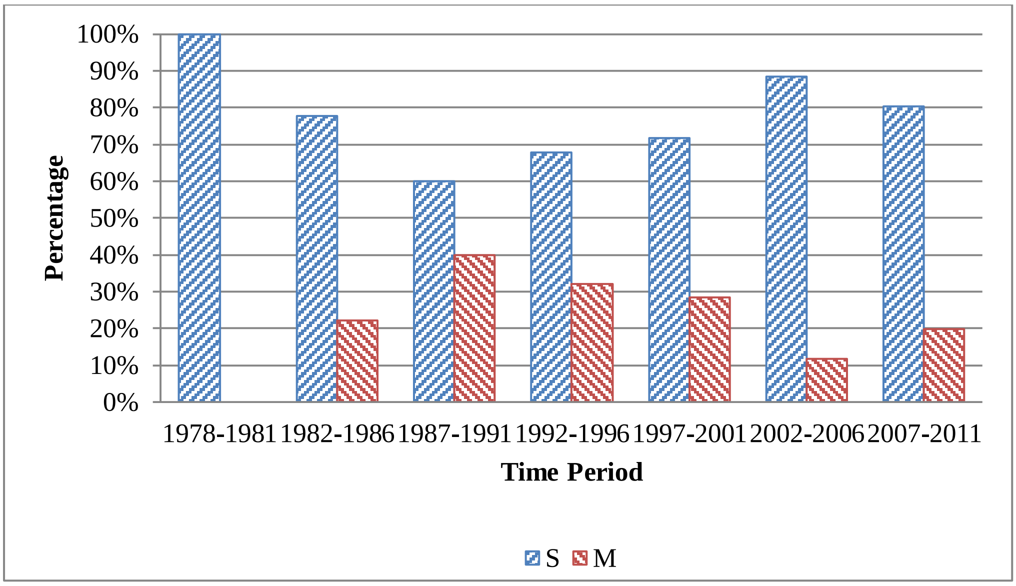

Figure 2-18. Share of studies by level of disaggregation over time

2.4.7 Cross-Country IDA Studies

We also review the cross-country studies in IDA. Cross-country decomposition studies allow analysts and decision-makers to have a better understanding of the underlying causes of variation in energy-related aggregate between countries (Zhang and Ang, 2001).

In this survey, thirteen studies deal with cross-country decomposition. Eight studies apply intensity indicator, three studies use quantity indicator, and two studies adopt both intensity and quantity indicators. Intensity indicator is preferred as it helps to eliminate the influence of economy or population size. Nine studies deal with energy-related emissions and four studies concern energy consumption. It is found that researchers are more interested in energy-related emission comparisons among countries. Nine studies have only one level of disaggregation. Among them, seven treat the economy as a whole and have no disaggregation. Only four studies have two levels of disaggregation. The reason is that in international comparisons, the data has to be consistent, and level of disaggregation and data compatibility are potential areas of difficulty.

2.4.8 Two-Dimensional Analysis

The breakdown of the surveyed studies by treatment of time and indicator type between 1978 and 2011 is shown in Table 2-1. The total counts for all indicator types for both chaining and non-chaining approaches are more than the total number of studies because some studies dealt with more than one indicator type or more than one approach to treat time.

The last row of Table 2-1 gives the total counts for chaining and non-chaining approaches, and non-chaining is adopted more than chaining approach. From this table, we find that quantity indicator with non-chaining approach is most often adopted in IDA studies. The reasons may be that quantity indicator is easier to interpret compared to ratio indicators and non-chaining is traditionally adopted more often than chaining approach as shown in Table 2-1. In application, for quantity indicator, non-chaining approach is adopted more often than chaining approach, while for ratio indicator and other indicators, chaining and non-chaining have the similar popularity.

The breakdown of the surveyed studies by treatment of time and decomposition method between 1978 and 2011 is shown in Table 2-2. The total counts for all decomposition methods for both chaining and non-chaining approaches are more than the total number of studies because some studies dealt with more than one decomposition method or more than one approach to treat time.

Table 2-1.Number of studies by treatment of time and indicator type between 1978 and 2011

Treatment of time / Indicator type	Chaining	Non-chaining	Total
Quantity	71	105	176
Ratio	59	56	115
Others	3	2	5
Total	133	163	296

From Table 2-2, we see that when Laspeyres-based methods are adopted, non-chaining approach is more often used compared to chaining approach. When Divisia-based methods are adopted, chaining approach is more often used compared to non-chaining approach. The reason may be that the concepts of Divisia methods have a close relationship with chaining approach (details will be discussed in Chapter 6).

Table 2-2. Number of studies by treatment of time and decomposition method between 1978 and 2011

Treatment of time / IDA method	Chaining	Non-chaining	Total
Laspeyres-based	48	93	141
Divisia-based	86	68	154
Others	1	3	4
Total	135	164	299

The breakdown of the surveyed studies by decomposition approach and indicator type between 1978 and 2011 is shown in Table 2-3. Again, the total counts for all indicator types for both additive and multiplicative approaches are more than the total number of studies because some studies dealt with more than one indicator type or more than one decomposition approach.

From Table 2-3, we can see that quantity indicator with additive decomposition approach is most often adopted in IDA studies. For ratio indicator and other indicators, additive and multiplicative approaches have a balanced popularity.

Table 2-3. Number of studies by decomposition approach and indicator type between 1978 and 2011

Decomposition approach / Indicator type	Multiplicative	Additive	Total
Quantity	57	144	201
Ratio	74	70	144
Others	4	4	8
Total	135	218	353

2.5 Summary for Literature Review

In this chapter, we reviewed 280 IDA studies in both methodology and application aspects and aimed to help researchers and practitioners to have a better idea about the IDA literature. From this review, we found that with the worldwide concern about global warming, the main application area has shifted from energy studies to the emission studies. In addition, with the extensive development and wide acceptance of IDA, the main application sector has shifted from industry to economy-wide.

Ang and Zhang (2000) conclude that the number of studies using quantity indicator was about the same as that using ratio indicator. In our survey, it is found that quantity indicator is becoming more popular. The reason may be that quantity indicators are easier to interpret compared to ratio indicators. Moreover, it is found that the additive approach has always been more widely used than the multiplicative approach.

Pertaining to the decomposition method aspect, the dominant methods have shifted change from the Laspeyres-based methods to the Divisia-based methods. S/S has become the most popular Laspeyres-based method since it was developed in 1998 (Sun, 1998) and LMDI has become the most popular Divisia-based method since it was first presented in 1997 (Ang and Choi, 1997).

Some research gaps have been identified from this survey. From the review of the development of IDA, it is found that IDA and INP have a close relationship in terms of concepts and properties. Although IDA has been used for more than 30 years, there is still a lack of studies summarizing the similarities and the differences between IDA and INP. This research gap is studied in Chapter 3. Many IDA methods have been developed by energy researchers and analysts since the 1970s, and it is significant to consolidate the fast growing number of methods into a unified decomposition framework. Chapter 4 and 5 will further study the properties and linkages of some commonly used IDA methods to generalize different IDA methods. Furthermore, this review helps to illustrate that in the treatment of time, chaining and non-chaining approaches, have almost been equally applied in recent years. However, there is still a lack of a rigorous study that compares the two approaches. In Chapter 6, the advantages and disadvantages of the approaches are discussed and recommendations are provided.

Table 2-4. Summary of decomposition studies and their specific features

No.	Study	Year	Country	Application Area									Indicator Type			Level of Disaggregation			Treatment of Time		Decomposition Scheme								
				Energy			GHG Emissions			Oth											Approach		Method						
																							Laspeyres			Divisia			Oth
				Eco	Ind	Oth	Eco	Ind	Oth	Eco	Ind	Oth	Q	R/I	Oth	N	S	M	Cha	Non-cha	Mul	Add	Las	S/S	Oth	LMDI	AMDI	Oth	
1	Myers and Nakamura	1978	USA		x									x		2	x			x		x	x						
2	Bossanyi	1979	UK		x									x		35	x			x		x	x						
3	Ostblom	1982	Sweden	x										x		24	x			x		x	x						
4	Thomas and Mackerron	1982	UK		x								x			7	x			x		x	x						
5	Hankinson and Rhys	1983	UK		x								x			20	x			x		x			x				
6	Hirst et al.	1983	US		x								x			472	x		x			x	x						
7	Jenne and Cattell	1983	UK		x									x		104		2		x		x	x						
8	Jenne and Cattell	1983	UK		x									x		125		2		x		x	x						
9	Marlay	1984	US		x								x			472	x		x			x			x				
10	Gowdy and Miller	1985	US		x								x			39	x			x		x	x						
11	Sterner	1985	Mexico		x								x			222	x			x		x			x				
12	Ang	1987	Singapore		x									x		28		2		x		x	x						
			Taiwan		x									x		14	x			x		x	x						
13	Bending et al.	1987	UK	x										x		4	x			x		x			x				
					x									x		9	x			x		x			x				
			UK, Germany, Italy, France	x										x		2	x			x		x			x				
					x									x		7	x			x		x			x				
14	Boyd et al.	1987	US		x									x		13		3	x		x						x		
15	Morovie et al.	1987	EU-10		x								x	x		11		2		x		x	x						

16	Reitler et al.	1987			x								x			2	x			x		x			x				
17	Boyd et al.	1988	US		x								x			8	x		x			x					x		
18	Doblin	1988	US		x									x		14	x			x	x		x		x				
19	Howarth	1991	USA		x								x			7	x		x		x		x						
#20	Huntington	1989	US		x									x		6	x			x	x						x		
21	Morovie et al.	1989	EU-12		x								x	x		11		2		x		x	x						
22	Li et al.	1990	Taiwan		x									x		17		2	x		x						x		
23	Schipper et al.	1990	US	x									x			33		2		x		x	x						
24	Howarth et al.	1991	8 OECD countries		x								x	x		6	x			x	x		x				x		
25	Torvanger	1991	9 OECD countries					x						x		6	x		x		x						x		
26	Ang et al.	1992	Singapore		x								x			28		8		x		x			x		x		
27	Lin	1992	China		x								x	x		15	x		x			x	x						
28	Liu et al.	1992	Taiwan		x								x			15	x			x		x					x		
29	Liu et al.	1992	Singapore		x								x			28	x			x		x	x		x		x	x	
30	Meyers and Schipper	1992	World countries	x									x	x		14		2		x		x	x						
31	Park	1992	republic of Korea		x								x			9	x			x		x	x						
32	Raggi and Barbiroli	1992	Italy	x									x			28	x			x		x			x				
33	Ang	1993	Singapore		x								x	x		28		5		x	x	x	x		x		x		
34	Howarth et al.	1993	Five OECD countries	x										x		21		2		x		x	x						
35	Huang	1993	China		x									x		6		2	x		x						x		
36	Gardner	1993	Ontario, Canada		x									x		21		2	x		x						x		
37	Grubler	1993	World regions			x			x				x	x		n.s.	x			x		x	x						
38	Park et al.	1993	26 countries		x								x			11	x			x		x	x						
39	Patterson	1993	New Zealand	x										x		20		2	x			x	x						
40	Ang and Skea	1994	UK		x								x	x		173		13		x		x	x				x		
41	Ang	1994	Singapore		x									x		28	x		x	x	x	x	x		x		x	x	

			Taiwan		x									x		17	x		x	x	x	x	x		x		x	x	
42	Ang and Lee	1994	Singapore		x								x			28	x		x	x		x	x		x		x	x	
			Taiwan		x								x			17	x		x	x		x	x		x		x	x	
43	Meyers et al.	1994	Poland		x									x		9	x			x		x	x						
44	Schipper	1994	World countries	x									x	x		n.s.				x		x	x						
45	Sinton and Levine	1994	China		x									x		267	x		x			x	x				x		
46	Wilson et al.	1994	Australia	x										x		n.s.	x		x			x	x						
47	Ang	1995	Singapore		x								x	x		28		5	x	x	x	x					x		
#48	Ang	1995	--		x								x	x	x	2	x		x	x	x	x	x		x		x	x	
49	Choi et al.	1995	South Korea		x									x		9	x		x		x						x		
50	Ang and Lee	1996	Singapore		x										x	28	x		x	x	x	x	x		x			x	
			Taiwan		x										x	17	x		x	x		x	x		x				
51	Golove and Schipper	1996	US		x			x					x			7	x			x		x	x						
52	Scholl et al.	1996	9 OECD countries						x				x			5	x			x		x	x						
53	Shrestha and Timilsina	1996	12 Asian countries						x					x		4	x		x		x						x		
54	Zhang and Folmer	1996	China				x						x			1	x		n.s.			x						x	
55	Ang and Choi	1997	Korea		x			x						x		9	x			x	x					x			
56	Ang and Pandiyan	1997	China					x						x		8	x		x		x						x	x	
			South Korea					x						x		9	x		x		x						x	x	
			Taiwan					x						x		17	x		x		x						x	x	
57	Bosseboeuf and Richard	1997	France	x									x			4	x			x		x			x				
58	Eichhammer and Mannsbart	1997	EU-12		x								x			19		2		x		x	x						
59	Farla et al.	1997	8 OECD countries		x								x			10	x		x			x					x		
60	Golove and Schipper	1997	US	x			x						x			n.s.		2		x		x	x						
61	Greening et al.	1997	10 OECD countries		x									x		7	x		x	x	x							x	

62	Haas	1997	10 OECD countries			x							x			n.s.	x			x	x		x						
63	Han and Chatterjee	1997	9 developing countries				x						x			21		2		x		x	x						
64	Lakshmanan and Han	1997	US			x			x				x			19		2		x		x	x						
65	Landwehr and Jochem	1997	West Germany	x									x			n.s.		2		x		x	x						
C66	Nagata	1997	US and Japan	x										x		n.s.		2	--	--		x	x						
67	Schipper et al.	1997	10 industrialized countries (OECD countries)				x						x			22		2		x		x	x						
68	Sheinbaum and Rodriguez	1997	Mexico					x					x			8	x			x		x	x						
69	Shrestha and Timilsina	1997	12 Asian countries						x					x		3	x		x		x						x		
C70	Theriault and Sahi	1997	10 OECD countries		x									x		6	x		--	--		x	x						
71	Worell et al.	1997	Seven countries (Brazil, China, France, Germany, Japan, Poland and US)		x								x			3	x		x			x					x		
72	Ang et al.	1998	Singapore		x								x			28	x			x		x				x			
			China					x					x			8	x			x		x				x			
			South Korea						x				x			5	x			x		x				x			
73	Farla et al.	1998	Netherlands	x										x		18		2	x			x			x				
74	Gardner and Elkhafif	1998	Ontario, Canada		x									x		21	x		x		x						x		
75	Greening et al.	1998	10 OECD countries					x						x		7	x		x		x							x	
76	Krackeler et al.	1998	13 OECD countries						x				x			1	x			x		x	x						
77	Lai et al.	1998	--								x			x		8	x		x		x						x		
78	Sheinbaum and Ozawa	1998	Mexico		x			x					x			1	x			x		x	x						
79	Shrestha and Timilsina	1998	South Korea and Thailand						x					x		10		2	x		x						x		
80	Sun	1998	World regions	x									x	x		1	x			x		x		x					

81	Sun	1998	China	x									x	x		6		2	x			x		x					
82	Sun and Malaska	1998	Developed countries				x							x		1	x			x		x		x					
#83	Ang	1999	--				x	x					x	x							x	x							
C84	Ang and Zhang	1999	3 OECD regions and 3 world regions				x						x	x		1	x		--	--		x				x			
85	Greening et al.	1999	10 OECD countries						x					x		4	x		x		x							x	
86	Sun	1999	OECD region				x						x			1	x		x			x		x					
87	Unander et al.	1999	13 OECD countries		x								x			7	x		x		x							x	
88	Viguier	1999	3 Eastern countries and 3 OECD countries				x							x		3	x			x	x						x		
89	Zarnikau	1999	US		x								x	x		19	x			x	x	x					x	x	
#90	Ang and Zhang	2000	--	x	x	x	x	x	x	x	x	x	x	x	x						x	x	x	x	x	x	x	x	
91	Ang et al.	2000	--								x			x		8	x		x		x	x				x			
											x			x		254	x		x		x	x				x			
92	Farlar and Blok	2000	Netherlands	x									x			21		2		x	x				x				
93	Farlar and Blok	2000	Netherlands	x									x			21		2	x			x			x				
94	Hoffrén et al.	2000	Finland							x			x			9	x		n.s.			x		x					
95	Jung and Park	2000	Korea		x									x		5	x		x		x						x		
96	Liaskas et al.	2000	EU-13					x					x			n.s.	x			x		x	x						
97	Mazzarino	2000	Italy						x				x			9		2		x		x	x						
98	Nag and Kulshreshtha	2000	India				x							x		4	x		x		x						x		
			India						x					x		6	x		x		x						x		
99	Nag and Parikh	2000	India				x							x		4	x		x		x						x		
100	Shorrock	2000	UK						x				x			1	x		x			x			x				
C101	Sun	2000	EU-15				x							x		1	x		--	--		x		x					
C102	Sun	2000	Finland and Sweden				x							x		1	x		--	--		x		x					

103	Sun and Ang	2000	EU-15				x						x			1	x			x		x		x					
104	Ang and Liu	2001	China					x					x			8		2		x	x					x			
			World				x						x			8		2		x	x					x			
105	Choi and Ang	2001	Korea				x							x		4	x		x		x					x			
106	Greening et al.	2001	10 OECD countries						x					x		6	x		x		x							x	
107	Murtishaw and Schipper	2001	US	x									x			28		2	x		x							x	
108	Murtishaw et al.	2001	8 IEA countries		x			x					x	x		3	x		x		x							x	
C109	Schipper et al.	2001	IEA-14				x							x		27		2	--	--	x		x						
110	Schipper et al.	2001	14 IEA	x			x						x			16		2		x	x		x						
C			IEA-13		x									x		7	x		--	--		x	x						
111	Schipper et al.	2001	13 IEA countries					x					x			9	x		x		x							x	
112	Schleich et al.	2001	Germany				x						x			1	x			x		x		x					
113	Stage	2001	Namibian	x										x		17	x			x	x					x			
114	Sun	2001	15 EU countries	x									x			1	x		x			x		x					
C115	Zhang and Ang	2001	OECD, FSU/CEE, ROW				x						x			1	x		--	--		x				x			
116	Albrecht et al.	2002	Belgium, France, Germany, UK				x						x			3	x			x		x		x					
117	Ang and Choi	2002	Korea						x					x		6	x		x		x					x			
118	Choi and Ang	2002	Korea			x								x		4	x		x		x					x			
119	Davis et al.	2002	US				x							x		1	x		x		x						x		
120	Farla and Blok	2002	Netherlands		x								x			30		2		x	x				x				
121	Hamilton and Turton	2002	OECD countries				x						x			3	x			x	x		x						
122	Kim and Worrell	2002	Brazil, China, South Korea, US					x					x	x		3	x		x			x			x				
123	Kim and Worrell	2002	Korea, Mexico, Brazil,China, India, US					x					x	x		6	x		x			x			x				
124	Luukkanen and	2002	Nordic	x			x						x			1	x			x		x		x					

	Kaivo-oja2002		countries: Finland, Denmark, Norway and Sweden																										
125	Luukkanen and Kaivo-oja	2002	ASEAN countries	x			x						x			1	x			x		x		x					
126	Luukkanen and Kaivo-oja	2002	key developing contries	x			x						x			1	x			x		x		x					
127	Nanduri et al.	2002	Canada		x								x			26		3		x		x							x
128	Ozawa et al.	2002	Mexico		x			x					x			6	x		x			x			x				
129	Wade	2002	US	x									x			73		2	x		x						x		
C130	Ang et al.	2003	OECD and the developing world				x						x			1	x		--	--		x	x	x		x			x
131	Choi and Ang	2003	Singapore		x									x		28	x		x		x	x				x			
			Taiwan		x									x		17	x		x		x	x				x			
132	Hoekstra and van den Bergh	2003		x	x	x	x	x	x	x	x	x	x	x	x	2	x		x		x	x	x	x	x	x	x	x	
133	Gonzalez and Suarez	2003	Spain		x									x		28		2	x		x					x	x	x	
134	Liu and Ang	2003	--		x									x		--	--			x	x	x	x	x	x	x	x		x
135	Sun	2003	Finland	x			x						x			1	x		x			x		x					
136	Zhang	2003	China		x								x			29	x			x		x			x				
137	Ang	2004		x			x			x			x	x		2	x		x	x	x	x	x			x	x		
138	Ang	2004		x			x			x			x	x		2	x			x	x	x				x			
139	Ang et al.	2004	Korea				x						x			7	x			x	x				x				
140	Bhattacharyya and Ussanarassamee	2004	Thailand		x			x						x		11	x			x	x					x			
141	Boyd and Roop	2004	US		x									x		19	x		x		x				x				
142	Cornillie and Fankhauser	2004	22 transition countries	x										x		3	x		n.s.			x					x		
143	Fisher-Vanden et al.	2004	China		x									x		2582		6		x	x						x		
144	Greening et al.	2004	10 OECD countries						x					x		6	x		x		x						x		
145	Kaivo-oja and Luukkanen	2004	EU-15 and Norway	x			x						x			1	x			x		x		x					

146	Paul and Bhattacharya	2004	India				x						x			4	x			x		x		x					
147	Sun	2004	OECD				x						x			1	x			x		x		x					
148	Unander et al.	2004	Denmark, Norway, Sweden			x								x		5	x			x	x		x						
149	Aguayo and Gallagher	2005	Mexico		x									x		29	x			x		x			x				
150	Ang	2005	Canada		x			x					x			23	x			x	x	x				x			
151	Bhattacharyya and Ussanarassamee	2005	Thailand		x									x		11	x			x	x					x			
152	Cole et al.	2005	European					x						x		22	x		x			x				x			
153	He	2005	China				x						x			32		2		x		x					x		
154	Jalas	2005	Finland			x							x			153		2		x		x	x						
155	Kwon	2005	Great Britain						x				x	x		2	x		x		x	x				x			
156	Kwon and Preston	2005	UK									x		x		3	x			x		x				x			
157	Nassen and Holmberg	2005	Sweden						x				x			2	x			x		x						x	
158	Nag and Parikh	2005	India						x					x		7	x		x		x						x		
159	Ramirez et al.	2005	Netherlands		x								x	x		55	x		n.s.		x					x			
160	Schafer	2005	7 world regions	x										x		4	x			x	x						x		
161	Ussanarassamee and Bhattacharyya	2005	Thailand		x								x			11	x			x		x	x						
162	Wang et al.	2005	China				x						x			1	x		x			x				x			
163	Wu et al.	2005	China				x						x			6	x		x		x					x			
164	Ang	2006		x									x			4	x			x	x					x			
165	Boonekamp	2006	--	x	x	x							x			--	--	--			x	x							
166	Dachraoui and Harchaoui	2006	Canada				x						x	x		47	x		x		x						x		
167	Diakoulaki et al.	2006	Greece				x						x			7	x			x		x		x					
168	Ebohon and Ikeme	2006	sub-Saharan African				x							x		1	x			x		x		x					
169	Ediger and Huvaz	2006	Turkey	x									x			3	x		x			x				x			

170	Geller et al.	2006	11 OECD countries	x										x		30	x			x		x	x						
171	Lee and Oh	2006	15 APEC countries				x						x			1	x			x		x				x			
C			15 APEC countries				x						x	x		1	x		--	--		x				x			
172	Lenzen	2006														1	x			x		x							x
173	Lin et al.	2006	Taiwan					x					x			22	x			x		x					x		
174	Lise	2006	Turkey				x						x	x		4	x		x			x		x					
175	Luyanga et al.	2006	Namibia							x				x		22	x			x	x					x	x		
176	Ramirez and Worrell	2006	World		x								x			8	x			x	x					x			
177	Steenhof	2006	China		x								x			37	x			x	x	x			x				
178	Steenhof et al.	2006	Canada						x				x			6	x			x	x		x						
179	Wood and Lenzen	2006	Australia				x						x			n.s.				x		x				x			
180	Wu et al.	2006	China				x						x			1	x			x		x				x			
181	Ang and Liu	2007	US		x								x			21	x		x	x	x		x		x	x	x		
			US			x							x			5	x		x	x	x		x		x	x	x		
			US			x							x			9	x		x	x	x		x		x	x	x		
182	Ang and Liu	2007	Canada					x					x			23	x			x		x				x			
			Korea						x				x			5	x			x		x				x			
183	Ang and Liu	2007	Korea					x					x			7	x			x		x				x			
C184	Bataille et al.	2007	G7 Nations				x							x		19		2	--	--		x				x			
185	Diakoulaki and Mandaraka	2007	14 EU					x					x			10	x			x		x		x					
186	Fan et al.	2007	China				x							x		3	x			x	x							x	
187	Greening et al.	2007			x								x																
188	Liao et al.	2007	China		x									x		36	x			x	x	x				x	x		
189	Liu et al.	2007	China					x					x			36	x		x			x				x			
#190	Liu and Ang	2007	--		x								x	x		--	--	--			x	x	x	x	x	x	x	x	x
191	Lu et al.	2007	Taiwan, Germany,						x				x			1	x			x		x					x		

			Japan and South Korea																										
192	Shrestha et al.	2007	Thailand				x						x			1	x			x		x		x					
193	Steenhof	2007	China						x					x		1	x		x		x		x						
194	Unander	2007	10 IEA countries		x								x			7	x			x	x		x						
195	Vera and Langlois	2007	Thailand	x									x			n.s.			x			x				x			
196	Al-Ghandoor et al.	2008	US		x									x		2	x		x			x						x	
197	Al-Ghandoor et al.	2008	US		x									x		2	x		x	x		x	x	x			x	x	
198	Bor	2008	Taiwan	x									x			20		3	x		x				x				
199	Liang and Zhou	2008	China					x						x		39	x		n.s.		x					x			
200	Fernandez and Fernandez	2008	EU-15				x						x			1	x		x	x		x		x					x
201	Hashimoto et al.	2008	Japan							x				x		20		2		x		x		x					
202	Hatzigeorgiou et al.	2008	Greece				x						x			1	x		x	x		x				x	x		
203	Korppoo et al.	2008	North-West of Russia		x								x			9	x			x		x		x					
204	Lescaroux	2008	US		x									x		19	x		x		x							x	
205	Lin et al.	2008	China	x										x		3	x		x			x	x						
			China		x									x		9	x		x			x	x						
206	Ma and Stern	2008	China				x						x			1	x		x		x					x			
207	Ma and Stern	2008	China	x										x		34		3	x			x				x			
208	Metcalf	2008	US	x										x		4	x			x	x				x				
209	Achao and Schaeffer	2009	Brazil			x							x			5	x		x			x				x			
			Brazil			x							x			2	x		x			x				x			
210	Al-Ghandoor et al.	2009	Jordan		x									x		7	x		n.s.			x		x					
211	Ang et al.	2009			x								x			2	x			x		x	x	x	x	x	x	x	
212	Arto et al.	2009	EU-15					x					x			9	x		x			x				x			
213	Boer	2009	Korea					x					x			7	x			x	x				x				
214	Dhakal	2009	China				x						x			1	x			x		x		x					

215	Kamakate and Schipper	2009	5 selected OECD countries: Australia, France, Japan, United Kingdom, United States			x							x			3	x			x	x		x						
216	Ipek Tunc et al.	2009	Turkey				x						x			3	x		x			x				x			
217	Malla	2009	seven Asia-Pacific and North American countries						x				x			3	x		x			x				x			
218	Mairet and Decellas	2009	France			x							x			42		2	x		x	x				x			
219	Papagiannaki and Diakoulaki	2009	Greece and Denmark						x				x			32		3	x			x				x			
220	Pardo Martinez	2009	Germany and Colombia		x								x	x		n.s.	x		n.s.		x					x			
221	Salta et al.	2009	Greece		x								x			46		2	x			x				x			
222	Shrestha et al.	2009	15 selected countries in Asia and the Pacific						x				x			3	x		x		x					x			
223	Sorrell	2009	UK			x							x			6	x		x	x	x					x			
224	Steenhof	2009	China						x					x		1	x		x		x		x						
225	Timilsina and Shrestha	2009	12 Asian countries						x				x			4	x		x		x	x				x			
226	Tol et al.	2009	US	x										x		3	x			x	x						x		
			US				x						x			5	x			x		x	x						
227	Weber	2009	US	x									x			398		4		x		x				x			
228	Zha et al.	2009	China		x									x		36	x			x	x					x	x		
229	Zhang et al.	2009	China				x						x	x		3	x		x			x		x					
230	Zhang et al.	2009	China				x						x			4	x			x		x		x					
231	Al-Ghandoor et al.	2010	Jordan		x								x			7	x		n.s.			x			x				
232	Ang et al.	2010	Singapore	x									x			40		2		x	x	x				x			
		2010	US		x								x			18	x		x	x	x	x				x			

233	Bhattacharyya and Blake	2010	Seven MENA (Middle East and North African) countries							x				x		1	x			x		x	x						
234	Bhattacharyya and Matsumura	2010	EU-15				x							x		1	x			x	x					x			
235	Bruneau and Renzetti	2010	Canada							x				x		145	x			x		x				x			
236	Cahill and Gallachoir	2010	Ireland		x								x			12	x		x		x		x		x	x	x		
			French		x								x			9	x		x		x		x		x	x	x		
			Germany		x								x			13	x		x		x		x		x	x	x		
237	Dong et al.	2010	Japan-China					x					x			24	x			x	x	x				x			
238	Fisher-Vanden and Ho	2010	China				x						x			33	x			x	x						x		
239	Hatzigeorgiou et al.	2010	EU-25 (as a whole) and Greece				x						x			1	x			x	x					x			
240	He	2010	China					x					x			13	x		x		x						x		
241	Henriques and Kander	2010	13 countries	x										x		6	x			x	x					x			
242	Huntington	2010	US	x										x		65		2		x	x				x				
243	Lofgren and Muller	2010	Sweden				x						x			46	x		x			x				x			
244	Ma	2010	China	x										x		31	x		x			x				x			
#245	Ma et al.	2010	China	x	x									x		--	--	--			x	x	x	x	x	x	x	x	
246	Mendiluce et al.	2010	Spain and EU 15	x										x		16		2		x		x				x			
C			Spain and EU 15	x										x		16		2	--	--		x				x			
247	Oh et al.	2010	South Korea				x						x			12		2	x			x				x			
248	Oguchi et al.	2010	Japan								x		x			--	--	--	x			x		x					
249	Pardo Martinez	2010	German and Colombia		x									x		9	x		n.s.		x					x			
250	Pani and Mukhopadhyay	2010	World				x						x			1	x		x			x				x			
251	Reddy and Ray	2010	India		x			x					x	x		13	x		x			x		x					
252	Sheinbaum et al.	2010	Mexico		x			x					x			5		2		x		x				x			

253	Sheinbaum et al.	2010	Five Latin American countries				x						x			5	x			x	x					x			
254	Tao	2010	China	x										x		1	x		x			x		x					
255	Taylor et al.	2010	IEA countries	x									x			24		2		x	x		x						
256	Vinuya et al.	2010	US				x						x			1	x			x		x				x			
257	Wang et al.	2010	China		x								x			39	x		x			x				x			
258	Zha et al.	2010	China						x				x			1	x		n.s.			x				x			
259	Zhao et al.	2010	China					x					x			33		2		x		x				x			
260	Zhao et al.	2010	China		x								x			15	x		x			x				x			
261	Akbostanci et al.	2011	Turkey					x					x			57	x		x			x				x			
262	Al-Mansour	2011	Slovenia	x									x			26		2	x		x				x				
263	Choi and Ang	2011	US		x									x		21	x		x		x					x			
264	Chung et al.	2011	Hong Kong			x							x			4	x		x			x				x			
265	de Freitas and Kaneko	2011	Brazil				x						x			6	x		x		x	x				x			
266	de Freitas and Kaneko	2011	Brazil				x						x			6	x		x			x				x			
C267	Gingrich et al.	2011	Austria and Czechoslovakia				x						x			1	x		--	--		x				x			
268	Hammond and Norman	2011	UK					x					x			20		2	x			x				x			
269	Inglesi-Lotz and Blignaut	2011	Africa	x									x			14	x		x			x				x			
270	Kumbaroglu	2011	Turkey				x						x			14		2	x			x		x					
271	Liu et al.	2011	China (Chengdu city)						x				x			7	x			x		x				x			
272	Mendiluce and Schipper	2011	Spain						x				x			5	x			x	x					x			
			Spain						x				x			5	x			x	x					x			
273	Pani and Mukhopadhyay	2011	World				x						x			1	x		x			x			x				
274	Sheinbaum et al.	2011	Five Latin American countries				x						x			5	x			x	x					x			
275	Stecket et al.	2011	China				x						x			1	x		n.s.			x		x					

276	Steenhof and Weber	2011	Canada						x				x			1	x		x			x	x						
277	Sun et al.	2011	China					x					x			1	x		x	x		x				x			
278	Tan et al.	2011	China				x							x		1	x			x		x				x			
279	Wang et al.	2011	China						x				x			5	x			x		x				x			
280	Zhang et al.	2011	China			x							x			10		2	x			x				x			
Total				55	100	16	73	37	34	9	5	3	184	128	4		218	64	119	148	116	191	77	43	43	96	61	27	6

Eco, Economy; Ind, Industry; Oth, Others; Q, Quantity; R/I, Ratio or index; N, Number; S, Single-level; M, Multi-level; Cha, Chaining; Non-cha, Non-chaining; Mul, Multiplicative; Add, Additive; Las, Laspeyres; S/S, Shapley/Sun;
Survey study
C Cross-country decomposition

CHAPTER 3: Index Decomposition Analysis and Index Number Problem

3.1 Introduction

As reviewed in Chapter 2, in the early stage of IDA studies, the decomposition formulae used by researchers were straightforward and intuitive. Researchers changed a specific variable while keeping others unchanged, in order to gauge the effect of this variable. This approach shared some principles with the Laspeyres index numbers in economics. Boyd et al. (1988) first pointed out that energy decomposition analysis is analogous to INP in economics. The authors also suggested that the IDA problem could call upon the index number literature and approach for guidance on both methods and properties. Additive Divisia decomposition was proposed, and the AMDI method was developed in that paper. Since then, many popular IDA methods have been derived from INP. For example, Ang and Choi (1997) and Ang and Liu (2001) proposed refined Divisia index methods: the multiplicative LMDI II method and the multiplicative LMDI I method, respectively, based on the study of Sato (1976) using INP.

Liu and Ang (2003) were the first to study the linkages between IDA and INP and introduced three IDA methods (Fisher ideal, Vartia I and Stuvel) derived from INP for the first time. In this Chapter, we will extend the studies of Boyd et al. (1988) and Liu and Ang (2003) to study the theoretical foundations of IDA from the viewpoint of INP in economics. Based on the discussion of similarities and differences between IDA and INP, we borrow some axioms and tests from INP and combine them with the tests developed independently based on IDA to provide a summary of criteria in method selection.

In the following sections, firstly, we introduce INP in Section 3.2, followed by discussions of the similarities and differences between IDA and INP in both methodological and application aspects in Section 3.3. In Section 3.4, axioms and tests for method selection are established to provide suggestions for researchers, corresponding to different situations and data sources. Recommendations and conclusions are presented in Section 3.5.

3.2 Introduction of INP

INP has a long history in economics, with some of the most important contributions from Edgeworth, Laspeyres, Paasche and Fisher et al. Methods proposed by Laspeyres (1871), Paasche (1874) and Fisher (1922) are still commonly used by some national statistical offices around the world.

3.2.1 Definition of Index Numbers

One classic definition of index numbers is from Edgeworth (1925) which defines an “index-number” as “a number adapted by its variations to indicate the increase or decrease of a magnitude not susceptible of accurate measurement”. The magnitude that Edgeworth had in mind was the general price level in economics or the value (purchasing power) of money that was based on non-observable quantity. Allen (1975) comments that “it makes no attempt to get a measure or indicator of the actual level attained by the non-observable magnitude.” An index number is calculated to measure changes in the magnitude from one situation to another. The two situations could be two time points (e.g. two different years), or two situations in a spatial sense (e.g. two counties), or other kinds of two situations. For the price index, it studies the change of general price level in economics and the purchasing power of money through time. The quantity index studies the change of quantities of goods and services (so-called real developments) over time in economics.

3.2.2 Approaches Used in INP

Balk (2008) summarizes different approaches to study price index and quantity index as: “test approach”, “economic approach” and “stochastic approach”. These three different approaches study price index and quantity index under different assumptions and using different theories. Balk (2008) comments that “axiomatic approach” or “test approach” has no formal economic theory involved and price and quantity are independent variables. “Economic approach” uses formal optimization models with utility function in economics. This means that price and quantity are interdependent variables. “Stochastic approach” uses stochastic model with assumptions of probability distributions of random variables.

Balk (2008) summarizes “test approach” in a systematic way as basket-type indices, geometric mean indices and unit value index. For basket-type indices, the price or quantity indices are expressed as weighted arithmetic or harmonic means of price or quantity relatives. Examples are the Laspeyres, Paasche and Marshall-Edgeworth price and quantity indices. For geometric mean indices, the price or quantity indices are defined as geometric means of price or quantity relatives. Examples are the Sato-Vartia price and Törnqvist quantity indices. For unit value index, it is introduced by Drobisch and the price index is the value ratio divided by the simple sum or Dutot quantity index.

Parallel to the development of price and quantity indices, it is significant to evaluate different price and quantity indices objectively. Fisher (1922) adopts several criteria (tests) to distinguish different price and quantity indices and concludes that Fisher's method is an "ideal" method.

Diewert et al. (2009) comments that "the absence of criteria for selecting the tests is matched by the absence of criteria for ranking the importance of the various tests, except by arbitrarily ranking all tests equally (contending, for example, that the Fisher index, or some other index, passes more tests that other indexes). The value of any index number property depends on the index number purpose, so no system of discriminating among tests, including equal weighting, has universal applicability". There are no price indices or quantity indices that satisfy all the tests. In addition, there are no clear guidelines to rank all the tests and therefore no unique evaluation for the different index number formulae.

There have been debates about the concepts of Consumer Price Index (CPI) and Cost-of-Living Index (COLI). CPI measures change of the general level of price, while, COLI would measure the change of cost for living with a constant standard of living. Bureau of Labor Statistics (2012) discusses the differences between CPI and COLI: "traditionally, the CPI was considered as upper bound to a COLI in that the CPI did not reflect the changes in buying or consumption patterns that consumers would make to adjust to relative price changes. The ability to substitute means that the increase in cost to consumers of maintaining their level of well-being tends to be somewhat less than the increase in the cost of the mix of goods and services they previously purchased." To get the standard of living constant, utility theory is used in "economic approach" to study the COLI. In addition, each consumer is assumed to be equipped with a well-behaved preference ordering, which can be represented by a utility function with a scalar utility level.

Diewert et al. (2009) comments that “more recently, the test approach has become popular in Europe, especially in academic circles, while the economic approach remains popular in the United States”. Balk (2008) comments that “the economic theory of index numbers soon became the dominant theory. Very important was the development and application of duality theory (which is concerned with the various representations of a preference ordering) and Diewert’s (1976) introduction of the concept of a superlative price index”. Although the “economic approach” is widely adopted to study index numbers, we only compare the similarities and differences between “test approach” in INP and IDA used in energy studies. The reason is that “economic approach” needs utility function in economics and IDA has no utility function in energy-related studies. In addition, when using “economic approach”, price and quantity are interdependent variables while the factors in IDA are assumed to be independent.

3.2.3 Formulae of Index Numbers

Index numbers could be proposed either by multiplicative (ratio) or additive (differences) approaches. Similar to the definitions of multiplicative and additive approaches discussed in Chapter 1, ratio change means index number is measured using the ratio of indices; and difference change means index number is measured using difference of indictors. In general, index numbers are measured in ratio approach.

For the general INP formulation, assume that there are N commodities. The price for commodity i is denoted by p_i and the quantity for commodity i is denoted by q_i, where $i = 1, \ldots, N$. Then, the value for commodity i is given by $v_i = p_i \cdot q_i$. The vector of prices is denoted by $p \equiv (p_1, \ldots, p_N)$, the vector of quantity is denoted by $q \equiv (q_1, \ldots, q_N)$. The price index for period T relative to period 0 is given by $P(p^T, q^T, p^0, q^0)$ and the quantity index for period T relative to period 0 is given by $Q(p^T, q^T, p^0, q^0)$, where

$$v^T / v^0 = P(p^T, q^T, p^0, q^0) Q(p^T, q^T, p^0, q^0) \tag{3-1}$$

The price indicator for period T relative to period 0 is given by $\mathcal{P}(p^T, q^T, p^0, q^0)$ and the quantity indicator for period T relative to period 0 is given by $\mathcal{Q}(p^T, q^T, p^0, q^0)$. The difference of value between period T relative to period 0 is given by $v^T - v^0$ and

$$v^T - v^0 = \mathcal{P}(p^T, q^T, p^0, q^0) + \mathcal{Q}(p^T, q^T, p^0, q^0) \tag{3-2}$$

3.3 Linkages and Differences between IDA and INP

In this section, linkages and differences between IDA and INP in both the methodological and application aspects are discussed.

3.3.1 Linkages between IDA and INP

IDA is a popular technique used in energy-related studies to quantify the impacts of various pre-defined factors to the total change of an energy-related aggregate. INP is used to study the decomposition of expenditure value change of economic flows into price and quantity indices or indicators. The objectives of both IDA and "test approach" in INP are the same: to decompose the change of aggregate indices or indicators into influencing components. In addition, "change of energy-relative aggregate” in IDA is similar to “expenditure value change of economic flows” in INP. The “pre-defined factors” in IDA is similar to the “price level and quantity level” in INP. More specifically, Liu and Ang (2003) presented that for 2-factor identity, the measure of the structural change of aggregate energy intensity may be seen as related to the quantity index, while that of sectoral intensity as related to the price index.

Secondly, the pre-defined factors influencing the total change of energy-related aggregate are assumed to be independent in IDA. The price level and quantity level are assumed to be independent for “test approach” in INP.

Thirdly, although index numbers are measured in multiplicative approach in general, both IDA and “test approach” in INP could be proposed either by multiplicative or additive approaches.

Fourthly, both IDA and index numbers are concerned with methods comparison and evaluation. Criteria (tests) are developed to find suitable methods that can satisfy an appropriate set of properties.

Finally, some popular IDA methods may have similar formulae as index numbers in INP and this is a significant linkage between INP and decomposition methodology. The summaries of INP and IDA methods are tabulated in Table 3-1 and Table 3-2 respectively. Since index number theory usually uses two factors only, we present IDA methods for decomposing the aggregate energy intensity ($I = \frac{E}{Y} = \sum_j \frac{E_j}{Y} = \sum_j \frac{Y_j}{Y} \cdot \frac{E_j}{Y_j} = \sum_j S_j \cdot I_j$) into structure and intensity effects in Table 3-2.

Laspeyres (1871) proposes the Laspeyres price index that measures changes over time by holding quantities at their base year value and by letting price variables change from year 0 to year *T*. Fisher (1922) shows that Laspeyres price index could be written as a weighted arithmetic mean of price relatives using the base period value shares as weights.

Paasche (1874) proposes the Paasche price index that measures changes over time by holding quantities at their target year value and by letting price variables change from base year to target year. And Marshall (1887) proposes the price index that measures changes over time by holding quantities as the average of their target year value and base year value and by letting price variables change from base year to target year. Edgeworth (1925) presents that this price index is preferred, which is now known as the Marshall-Edgeworth price index.

Fisher (1922) proposes the Fisher ideal index as the geometric mean of the Laspeyres and Paasche indices. And Bennet (1920) develops Bennet indicator as the arithmetic mean of the Laspeyres and Paasche indicators.

Törnqvist, Montgomery-Vartia, Sato-Vartia indices refer to the concept of Divisia integral index (Divisia, 1925) and they are quite distinct from the five indices or indicators mentioned above. These three indices differ by their different weights selected: Törnqvist (1936) uses simple arithmetic means of the base and target year value shares as weights for Törnqvist index; Montgomery-Vartia and Sato-Vartia indices use “logarithmic mean” weights given in Vartia (1974) and Sato (1976). Montgomery (1929 & 1937) develops the Montgomery indicator, which is the additive counterpart of the Montgomery-Vartia index.

In Table 3-2, the Laspeyres and S/S IDA methods are studied in Section 2.3.3 and AMDI, LMDI I, LMDI II IDA methods are studied in Section 2.3.4. The Paasche IDA method is similar to the Paasche index in concept and it has not been widely used. One early example of the Paasche IDA method is Doblin (1988). Marshall-Edgeworth IDA method is similar to the Marshall-Edgeworth index in concept and Reitler et al (1987) is one of the first studies that applied this method. Fisher ideal IDA method is first adopted in a 2-factor IDA study in Liu and Ang (2003) and this method is derived from the Fisher ideal index in INP. From Table 3-1 and Table 3-2, we could find that some popular IDA methods have formulae similar to index numbers in INP.

Table 3-1. Formulae for main index numbers

	$\mathcal{P}(p^T, q^T, p^0, q^0)$	$\mathcal{Q}(p^T, q^T, p^0, q^0)$	$P(p^T, q^T, p^0, q^0)$	$Q(p^T, q^T, p^0, q^0)$
Laspeyres	$\sum_i q_i^0 \cdot \Delta p_i$	$\sum_i p_i^0 \cdot \Delta q_i$	$\sum_i p_i^T \cdot q_i^0 / \sum_i p_i^0 \cdot q_i^0$	$\sum_i p_i^0 \cdot q_i^T / \sum_i p_i^0 \cdot q_i^0$
Paasche	$\sum_i q_i^T \cdot \Delta p_i$	$\sum_i p_i^T \cdot \Delta q_i$	$\sum_i p_i^T \cdot q_i^T / \sum_i p_i^0 \cdot q_i^T$	$\sum_i p_i^T \cdot q_i^T / \sum_i p_i^T \cdot q_i^0$
Marshall-Edgeworth Index			$\frac{\sum_i p_i^T \cdot (q_i^T + q_i^0)/2}{\sum_i p_i^0 \cdot (q_i^T + q_i^0)/2}$	$\frac{\sum_i q_i^T \cdot (p_i^T + p_i^0)/2}{\sum_i q_i^0 \cdot (p_i^T + p_i^0)/2}$
Fisher Index			$(\frac{\sum_i p_i^T \cdot q_i^0}{\sum_i p_i^0 \cdot q_i^0} \cdot \frac{\sum_i p_i^T \cdot q_i^T}{\sum_i p_i^0 \cdot q_i^T})^{0.5}$	$(\frac{\sum_i p_i^0 \cdot q_i^T}{\sum_i p_i^0 \cdot q_i^0} \cdot \frac{\sum_i p_i^T \cdot q_i^T}{\sum_i p_i^T \cdot q_i^0})^{0.5}$
Bennet Indicator	$\sum_i \frac{1}{2}(q_i^T + q_i^0) \cdot \Delta p_i$	$\sum_i \frac{1}{2}(p_i^T + p_i^0) \cdot \Delta q_i$		
Törnqvist Index			$\exp(\sum_i \frac{1}{2}(\frac{v_i^T}{v^T} + \frac{v_i^0}{v^0}) \cdot \ln(\frac{p_i^T}{p_i^0}))$	$\exp(\sum_i \frac{1}{2}(\frac{v_i^T}{v^T} + \frac{v_i^0}{v^0}) \cdot \ln(\frac{q_i^T}{q_i^0}))$
Montgomery Indicator	$\sum_i L(v_i^T, v_i^T) \cdot \ln(\frac{p_i^T}{p_i^0})$	$\sum_i L(v_i^T, v_i^T) \cdot \ln(\frac{q_i^T}{q_i^0})$		
Montgomery-Vartia Index			$\exp(\sum_i \frac{L(v_i^T, v_i^T)}{L(v^T, v^0)} \cdot \ln(\frac{p_i^T}{p_i^0}))$	$\exp(\sum_i \frac{L(v_i^T, v_i^T)}{L(v^T, v^0)} \cdot \ln(\frac{q_i^T}{q_i^0}))$
Sato-Vartia Index			$\exp(\sum_i \frac{L(\frac{v_i^T}{v^T}, \frac{v_i^0}{v^0})}{\sum_i L(\frac{v_i^T}{v^T}, \frac{v_i^0}{v^0})} \cdot \ln(\frac{p_i^T}{p_i^0}))$	$\exp(\sum_i \frac{L(\frac{v_i^T}{v^T}, \frac{v_i^0}{v^0})}{\sum_i L(\frac{v_i^T}{v^T}, \frac{v_i^0}{v^0})} \cdot \ln(\frac{q_i^T}{q_i^0}))$

Table 3-2. Formulae for main IDA methods

	ΔI_{int}	ΔI_{str}	D_{int}	D_{str}
Laspeyres	$\sum_j S_j^0 \cdot \Delta I_j$	$\sum_j I_j^0 \cdot \Delta S_j$	$\sum_j S_j^0 \cdot I_j^T \Big/ \sum_j S_j^0 \cdot I_j^0$	$\sum_j S_j^T \cdot I_j^0 \Big/ \sum_j S_j^0 \cdot I_j^0$
Paasche	$\sum_j S_j^T \cdot \Delta I_j$	$\sum_j I_j^T \cdot \Delta S_j$	$\sum_j S_j^T \cdot I_j^T \Big/ \sum_j S_j^T \cdot I_j^0$	$\sum_j S_j^T \cdot I_j^T \Big/ \sum_j S_j^0 \cdot I_j^T$
Marshall-Edgeworth	$\sum_j \frac{1}{2} \cdot (S_j^T + S_j^0) \cdot \Delta I_j$	$\sum_j \frac{1}{2} \cdot (I_j^T + I_j^0) \cdot \Delta S_j$	$\frac{\sum_j I_j^T \cdot (S_j^T + S_j^0)}{\sum_j I_j^0 \cdot (S_j^T + S_j^0)}$	$\frac{\sum_j S_j^T \cdot (I_j^T + I_j^0)}{\sum_j S_j^0 \cdot (I_j^T + I_j^0)}$
Fisher			$\left(\frac{\sum_j S_j^0 \cdot I_j^T}{\sum_j S_j^0 \cdot I_j^0} \cdot \frac{\sum_j S_j^T \cdot I_j^T}{\sum_j S_j^T \cdot I_j^0}\right)^{0.5}$	$\left(\frac{\sum_j S_j^T \cdot I_j^0}{\sum_j S_j^0 \cdot I_j^0} \cdot \frac{\sum_j S_j^T \cdot I_j^T}{\sum_j S_j^0 \cdot I_j^T}\right)^{0.5}$
S/S	$\sum_j S_j^0 \cdot \Delta I_j + \frac{1}{2} \cdot \sum_j \Delta S_j \cdot \Delta I_j$	$\sum_j I_j^0 \cdot \Delta S_j + \frac{1}{2} \cdot \sum_j \Delta S_j \cdot \Delta I_j$		
AMDI	$\sum_j \frac{1}{2} \cdot (\frac{E_j^T}{Y^T} + \frac{E_j^0}{Y^0}) \cdot \ln(\frac{I_j^T}{I_j^0})$	$\sum_j \frac{1}{2} \cdot (\frac{E_j^T}{Y^T} + \frac{E_j^0}{Y^0}) \cdot \ln(\frac{S_j^T}{S_j^0})$	$\exp(\sum_j \frac{1}{2} \cdot (\frac{E_j^T}{E^T} + \frac{E_j^0}{E^0}) \cdot \ln(\frac{I_j^T}{I_j^0}))$	$\exp(\sum_j \frac{1}{2} \cdot (\frac{E_j^T}{E^T} + \frac{E_j^0}{E^0}) \cdot \ln(\frac{S_j^T}{S_j^0}))$
LMDI I	$\sum_j L(\frac{E_j^T}{Y^T}, \frac{E_j^0}{Y^0}) \cdot \ln(\frac{I_j^T}{I_j^0})$	$\sum_j L(\frac{E_j^T}{Y^T}, \frac{E_j^0}{Y^0}) \cdot \ln(\frac{S_j^T}{S_j^0})$	$\exp(\sum_j \frac{L(\frac{E_j^T}{Y^T}, \frac{E_j^0}{Y^0})}{L(I^T, I^0)} \cdot \ln(\frac{I_j^T}{I_j^0}))$	$\exp(\sum_j \frac{L(\frac{E_j^T}{Y^T}, \frac{E_j^0}{Y^0})}{L(I^T, I^0)} \cdot \ln(\frac{S_j^T}{S_j^0}))$
LMDI II	$\sum_j \frac{L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})}{\sum_j L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})} \cdot L(I^T, I^0) \cdot \ln(\frac{I_j^T}{I_j^0})$	$\sum_j \frac{L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})}{\sum_j L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})} \cdot L(I^T, I^0) \cdot \ln(\frac{S_j^T}{S_j^0})$	$\exp(\sum_j \frac{L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})}{\sum_j L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})} \cdot \ln(\frac{I_j^T}{I_j^0}))$	$\exp(\sum_j \frac{L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})}{\sum_j L(\frac{E_j^T}{E^T}, \frac{E_j^0}{E^0})} \cdot \ln(\frac{S_j^T}{S_j^0}))$

3.3.2 Differences between IDA and INP

The application areas of INP and IDA are different. INP is studied in economics while IDA is applied in energy demand analysis, energy-related gas emission studies and other application areas. As discussed in Choi and Ang (2003): "methodologically, the underlying technique of IDA is linked to the INP in economics and statistics. Its development, however, has been driven by decomposition problems found in the energy and environment field". IDA has its special properties with respect to the energy field and owns its independent developments.

Data

In INP, price level (or quantity level) of different commodities are assumed to be independent in general. In IDA, when structural effect exists, the sum of the shares of sub-sectors should be equal to 1. This means that, the data of share are not independent among different sub-sectors.

In INP, the price or quantity indices are expressed as weighted arithmetic, harmonic or geometric means of price or quantity relatives. Therefore, different commodities are not required to have the same units. In IDA, when structural effect exists, there is a requirement that different sub-sectors should have the same unit to calculate the structural effect.

Number of Factors

In economics, there are only two indices: price index and quantity index. The number of decomposition effects in IDA studies is usually more than two. For instance, changes of energy consumption can be decomposed into activity effect, sectoral intensity effect and structure effect. In energy-related gas emission studies, changes of CO_2 emission can be decomposed into activity effect, fuel mix, energy intensity, structure, and CO_2 emission factor effects. The IDA methods derived from index numbers are usually needed to be extended from two factors to more than two factors cases.

IDA and INP Methods

Although some popular IDA methods were derived from INP, there are some IDA methods which were developed independently by decomposition problems found in energy analysis. Residual is a common problem needed explanation in IDA but not in INP. To deal with the residual problem, Sun (1998) proposes the S/S method based on the principle of "jointly created and equally distributed" of the residual. When the number of factors is two, this S/S method is similar to the Bennet indicator proposed in Bennet (1920). However, this IDA method is more general and is not derived from INP. Ang et al. (2004) fill the gap in IDA by extending the conventional two-factor Fisher index to *n* factors to complement an existing additive decomposition approach. Although this generalized Fisher method is similar to the method developed in Siegel (1945), it was derived using a different route and developed independently.

The properties and linkages of some IDA methods can be explained by decomposition problems found in the energy field. Ang et al. (2009) study the properties and linkages of some popular IDA methods in energy and carbon emission analyses. Specifically, the authors introduce a simple relationship among additive Divisia-based IDA methods and summarize a simple relationship among additive Laspeyres-based IDA methods. Those properties and linkages are rarely discussed in INP and are developed independently.

In summary, although some IDA methods were derived from index numbers, some IDA methods were developed independently and some linkages were studied among various additive Divisia-based IDA methods.

Additive and Multiplicative Approaches

From the literature review in Chapter 2, we can see that there have been more IDA studies using the additive approach compared with the multiplicative approach. The main reason is that additive approach has a good property of easy understanding.

In INP, both additive (difference) and multiplicative (ratio) approaches could be adopted. Traditionally, the ratio index is more widely used than the difference index number.

Zero and Negative Values

Liu et al. (1992b) point out the zero value problem for some fuel types in the data set, which in turn lead to computational problems in IDA methods. Chuang and Rhee (2001) point out the negative value problem of LMDI method in CO_2 emission decomposition. Several research studies discussed the zero and negative value problems. Examples are Ang and Choi (1997) and Ang and Liu (2007c). Driven by decomposition problems found in the energy field, the zero and negative problems have been well studied in IDA. However, this kind of problems is seldom studied in INP methods.

3.4 Criteria for IDA Methods

From the discussion of similarities and differences between IDA and INP in Section 3.3, we can draw two conclusions. One conclusion is that IDA is closely linked to INP, and some IDA methods and tests are derived from INP. The other conclusion is that IDA has its own development in both methods and tests aspects. Liu and Ang (2003) summarizes eight IDA methods derived from INP. In this section, we extend their work and provide a comprehensive study about criteria for evaluating IDA methods, including existing tests in IDA, new tests derived from INP and limitations of tests in IDA.

3.4.1 Existing Tests and Properties of IDA Methods

In this section, we review the existing tests and properties derived from INP and the tests developed independently based on the characteristics of IDA methods.

Factor reversal test

In INP, when $P(p^T, q^T, p^0, q^0) \cdot P(q^T, p^T, q^0, p^0) = p^T \cdot q^T / p^0 \cdot q^0$ is satisfied, a price index is called ideal or satisfying factor reversal test. There are two conditions to satisfy this test. The first one is that the product of price index and quantity index should equal to the change of value index. At the same time, the quality index has the same formula as price index with quantities and prices in the reverse order.

In IDA, factor reversal test suggests that the product of impacts of the pre-defined factors should equal to the ratio change of energy-related aggregate in multiplicative decomposition (the sum of impacts of the pre-defined factor should equal to the absolute change of energy-related aggregate in additive decomposition). In other words, the factor reversal test deals with the residual term issue, which leads to problem of interpretation

IDA methods satisfying this test are called perfect decomposition methods with no residual term. IDA methods that pass the factor reversal test will be preferred to those that generally give an unexplained residual term. In IDA, there is no emphasis on the consistency of formula for different factors. Some studies using different formula for different factors to pass the factor reversal test (for instance, Zhang (2003)).

In INP, when $P(p^T, q^T, p^0, q^0) \cdot Q(p^T, q^T, p^0, q^0) = p^T \cdot q^T / p^0 \cdot q^0$ is satisfied, a price index is called satisfying product test. For product test, there is no requirement that the quality index should have the same formula as price index with quantities and prices in the reverse order. From this point of view, the factor reversal test in IDA is actually linked to product test in INP.

From the discussions above, we recommend the factor reversal test in IDA should not only deal with residual issues, but also need to require the consistency of formula for different factors.

Time reversal test

In INP, when $P(p^T, q^T, p^0, q^0) = 1/P(p^0, q^0, p^T, q^T)$ is satisfied, a price is called satisfying time reversal test. This test requires that the price index from time *T* to Time 0 is the reciprocal of the price index from time 0 to Time *T*.

In IDA, the time reversal test is exactly the same as the definition in INP. The methods satisfying time reversal test means that the results calculated from basic year to target year are the reciprocal of the results calculated from target year to basic year in multiplicative decomposition (the results calculated from basic year to target year are the negative value of the results calculated from target year to basic year in additive decomposition). Passing this test means the methods are symmetric in time.

Consistency in aggregation

In INP, the concept of consistency in aggregation was developed by Vartia (1976) and formalized in Balk (1995) as shown below.

$$\psi(P_N(p^T, q^T, p^0, q^0), V^0, V^T) = \sum_{k=1}^{K} \psi(P_{N_k}(p_k^T, q_k^T, p_k^0, q_k^0), V_k^0, V_k^T)$$

Where, the set of commodities *A*={ $A_1, A_2, \ldots, A_N$ } is considered as a union of disjoint subsets A_k, $A = \bigcup_{k=1}^{K} A_k$; *N* denotes the number of commodities contained in *A*, N_k denote the number of commodities contained in A_k and $N = \sum_{k=1}^{K} N_k$; $(p_k^T, q_k^T, p_k^0, q_k^0)$ be sub-vector of $P(p^T, q^T, p^0, q^0)$

corresponding to the sub-aggregate A_k ; $\psi : \mathrm{R}^3_{++} \to \mathrm{R}_{++}$ is continuous and strictly increasing in its first argument.

Passing aggregation test enables users analyzing indicators either in one step or multiple steps with consistent results. There are two conditions for this test. The first one is that "the indices used in the single stage computation and those used in the first stage computation have the same functional form (only the numbers of variables are different)" (Balk, 1995). The other one is that "the formula used in the second stage computation has the same functional form as the indices used in the single and in the first stage after the following transformation has been applied: commodity indices are replaced by subaggregate indices and commodity values are replaced by subaggregate values" (Balk, 1995).

In IDA, consistency in aggregation test is derived from Vartia (1976) and Diewert (1978) in Ang and Liu (2001). In energy decomposition studies, it is common that decomposition is performed at two or more disaggregation levels and consistency in aggregation has been one of the most attractive properties in IDA.

Proportionality

In INP, when $P(\lambda\, p^T, q^T, p^0, q^0) = \lambda\, P(p^T, q^T, p^0, q^0)$ is satisfied, it passes proportionality test. This means that when the vector of price is multiplied by a common factor at the target year, the price index will be multiplied by this factor. In IDA, similar concept is defined: passing the proportionality test means that multiplication of a factor by a common factor leads to multiplication of the new decomposition results of this factor compared with the old decomposition results.

Special value robustness

As discussed in Section 3.3, INP has no problem in handling either zero or negative values in the dataset. In IDA, some methods related to Divisia index may behave badly when the dataset contains zero or negative values (for instance, AMDI method). The special value robustness test is concerned with whether complications would arise in the use of an IDA method when the data set contains zero or negative values. The IDA methods are special-value robust if they can give reasonable decomposition results for data set involving zero or negative value.

Additive/Multiplicative test

Passing the additive/multiplicative test means that the IDA methods could be adapted in both additive and multiplicative analysis and there is a direct and simple relation between additive and multiplicative formulae. The relationship allows the use of any of the two versions and yet the consistent results will be obtained for both additive and multiplicative decomposition.

Ease of use

Ease of use is a subjective concept and this criterion refers to the overall complexity of the formulae which define the IDA methods. In addition, as discussed in Section 3.3, there are generally more than 2 factors in IDA studies. IDA methods which can be easily extended from 2-factor case to more than two factor case are desirable.

3.4.2 "Partially" Fulfilled Problem

In the study above, we summarized the existing tests in IDA. In this part, we will discuss the problem arising from the existing tests in IDA.

As discussed in Section 3.3, in IDA, the sum of the shares of sub-sectors should be equal to 1 and the data of shares are not independent among different sub-sectors. With the limitation of structural data, some tests derived from INP cannot be satisfied in IDA and there are some problems in result interpretation.

For consistency in aggregation test, there is one condition that the values of factor used in the single stage computation and those used in the first stage of multi-step computation should have the same values. However, this condition can only be partially fulfilled, because some factors have different connotations at different levels of aggregation. Take two-factor case for example. Let N be the number of sectors, $n(i)$ is the number of sub-sectors for the particular sector i. E_{ij} represents the energy consumption for sub-sector j of sector i, Y_{ij} is the production level for sub-sector j of sector i, and I_{ij} represents energy intensity for sub-section j of sector i. Equations (3-3) and (3-4) show the formulation for one-step analysis and two-step decomposition analysis, respectively.

$$I^{one-step} = \sum_{i,j} I_{ij} = \sum_{i,j} \frac{Y_{ij}}{Y} \cdot \frac{E_{ij}}{Y_{ij}} = \sum_{i,j} S_{ij} \cdot I_{ij} \tag{3-3}$$

$$I_i^{two-step} = \sum_{j} I_{ij} = \sum_{j} \frac{Y_{ij}}{Y_i} \cdot \frac{E_{ij}}{Y_{ij}} = \sum_{j} S_{ij}^{'} \cdot I_{ij} \tag{3-4}$$

It can be clearly seen that since the expression of structural effect are different between them, the contribution of the factor derived from two-step analysis may not be the same as that from one-step analysis.

For proportionality test, take structure effect as an example. Since sum of the structure shares should be equal to 1, proportionality test is not meaningful for structure effect in energy decomposition studies.

These "Partially" fulfilled problems only arise for activity and share mix effects and energy intensity effect has no this kind of problems.

3.4.3 New Tests

Balk (1995) summarizes the axioms and tests used to evaluate index numbers in INP. Based on the study of Balk (1995), we borrow three new tests from INP to IDA.

Monotonicity test

When $P(p, q^T, p^0, q^0)$ is increasing in comparison period price p_n and $P(p^T, q^T, p, q^0)$ is decreasing in base period prices p_n $(n = 1, \ldots, N)$, price index passes monotonicity test. In IDA, the decomposition result of one factor should increase with respect to the target year value of that factor and should decrease with respect to the base year value of that factor.

Identity test

In INP, when all the comparison period prices are equal to the corresponding base period prices, then the price index number must be equal to 1: $P(p^0, q^T, p^0, q^0) = 1$. This means changing quantities will not affect the price index. In IDA, if the values of one factor keep constant in both target year and base year, the decomposition results of that factor should be equal to 1 in multiplicative analysis (be equal to 0 in additive analysis).

Circularity Test

In Fisher (1922), the so-called transitivity (circular) test was discussed in index numbers. For three time periods, (0,1), (1,2) and (0,2), this test requires that the price indices $P(p^2, x^2, p^1, x^1)P(p^1, x^1, p^0, x^0) = P(p^2, x^2, p^0, x^0)$. The price index number for period 2 relative to period 1 times the price index number for period 2 relative to period 0 is equal to the price index number for period 2 relative to period 0.

From the concept of transitivity test, we can see that in multiplicative decomposition, passing the transitivity test implies that chaining and non-chaining approaches produce the same results. We may extend the transitivity test to additive decomposition in IDA. When chaining and non-chaining approaches produce the same results for additive decomposition, we may conclude that the transitivity test is passed additively.

In Table 3-3, we summarize the tests in IDA to provide suggestions for related researchers to select IDA methods corresponding to different situations and data sources.

Table 3-3. Summary of tests in IDA

Tests Derived from INP	Circularity
	Consistency in aggregation
	Factor reversal test
	Identity
	Monotonicity
	Product test
	Proportionality
	Time reversal test
Tests Developed in IDA	Special value robustness
	Additive/Multiplicative decomposition
	Ease of use

3.5 Conclusions

In this chapter, we systematically study the linkages and the differences between IDA and INP, which is the theoretical foundation of the development of IDA. In addition, we summarize the existing tests to evaluate IDA methods, identify the problems of tests in IDA studies and introduced three new tests for the first time to better reflect whether a method is effective in performing decomposition analysis. From the literature review in Chapter 2, we could find that more and more methods have been proposed and applied in IDA studies. Different methods have different formulae which lead to different results. As a result, method selection for a specific research objective is essential. In many past decomposition studies, the choice of methods appeared to be arbitrary. Additionally, the most popular IDA methods are not necessarily the best in all situations. The summary of criteria will be helpful to researchers in the understanding and application of IDA methods corresponding to different situations and data sources.

CHAPTER 4: Laspeyres-based Index Decomposition Analysis Methods

4.1 Introduction

As introduced in the literature review in Chapter 2, the decomposition formulae used by researchers prior to the mid-1980s are straightforward and intuitive. The basic idea is the same as the Laspeyres method by isolating the impact of a certain variable to the change of an energy-related aggregate indicator by changing the impacting variable while holding other variables unchanged. The earliest studies using Laspeyres method include Bossanyi (1979) and Hankinson and Rhys (1983).

A similar concept is used to develop the Paasche method by letting the impacting variable change while holding other variables at the target year values. The earliest studies using the concept of Paasche include Reitler et al. (1987). For the M-E method, the mean of base and target year weights are used to calculate the impact of variables. Doblin (1988) is one of the earliest studies using the M-E method.

The Laspeyres method leaves a residual term, which is difficult to explain. Sun (1998) proposes a perfect method which is a refinement of the additive Laspeyres method. In the method, the residual is distributed equally among the main effects based on the "jointly created and equally distributed" principle. Liu and Ang (2003) develop the Fisher Ideal method (geometric means of the

Laspeyres method and the Paasche method) in IDA for the 2-factor case. Ang (2004) extends the Fisher Ideal method to the generalized Fisher method to study IDA for n-factor cases.

The IDA methods discussed above have similar concepts and we refer to this kind of IDA methods as Laspeyres-based methods.

Albrecht et al. (2002) are the first to introduce the Shapley value to energy IDA studies. Ang et al. (2003) later shows that the Shapley decomposition technique used by Albrecht et al. (2002) is the same as the method proposed independently by Sun (1998). Since they produce the same decomposition results, both methods are called the Shapley/Sun method in Ang (2004). Neither Albrecht et al. (2002) nor Ang et al. (2003), however, provides a formal treatment to the Shapley value in the IDA context. In this study, we shall formalize the relationships between methods linked to the Laspeyres index and the Shapley value through defining the characteristic function in the Shapley value.

The next section gives an introduction to Laspeyres-based IDA methods including the general formulae. It is followed by an introduction of the Shapley values in IDA in Section 4.3. The relationship between Laspeyres-based IDA methods and the Shapley value is discussed in Section 4.4. The conclusion is presented in Section 4.5.

4.2 Formulae of Laspeyres-based IDA Methods

In this section, we present the general formulae of Laspeyres-based IDA methods in both the additive and multiplicative approaches.

4.2.1 Additive Laspeyres-based IDA Methods

We study the general formula of additive Laspeyres-based IDA methods, based on the general IDA formula for *n* factors and *m* sub-categories used in Section 2.3:

$$\Delta V_{x_i} = \sum_{j=1}^{m}[\frac{V^0}{x_i^0} + \alpha(\frac{V^T}{x_i^T} - \frac{V^0}{x_i^0})] \cdot (x_i^T - x_i^0) \tag{4-1}$$

where $0 \leq \alpha \leq 1$. We have the additive Laspeyres method when $\alpha = 0$, the Paasche method when $\alpha = 1$, and the M-E method when $\alpha = 0.5$. These and other related IDA methods are referred to as "methods linked to the Laspeyres index" in the additive form in Ang (2004).

It is well-known that the application of the Laspeyres method in energy IDA studies leaves a residual term. This residual term was treated in a more formal way in Park (1992). Park (1992) decomposes changes in energy consumption in industry as defined in Eq. (4-2) and (4-4) and the effects are estimated by using the following Laspeyres index formulae:

$$\Delta E_{act} = \sum_{j}(Y^T - Y^0)S_j^0 I_j^0 \tag{4-2}$$

$$\Delta E_{str} = \sum_j Y^0 (S_j^T - S_j^0) I_j^0 \tag{4-3}$$

$$\Delta E_{int} = \sum_j Y^0 S_j^0 (I_j^T - I_j^0) \tag{4-4}$$

It further breaks down the residual term into four combinatorial product terms, the joint effects of changes in activity and energy intensity, activity and structure, energy intensity and structural change, and of all three variables:

$$\begin{aligned}\Delta E_{rsd} &= \sum_j (Y^T - Y^0)(I_j^T - I_j^0) S_j^0 + \sum_j (Y^T - Y^0)(S_j^T - S_j^0) I_j^0 \\ &+ \sum_j (I_j^T - I_j^0)(S_j^T - S_j^0) Y^0 + \sum_j (Y^T - Y^0)(S_j^T - S_j^0)(I_j^T - I_j^0)\end{aligned} \tag{4-5}$$

Many studies have shown that in energy IDA studies, the residual term ΔE_{rsd} given by the Laspeyres method can be very large. Furthermore, the combinatorial product terms such as those given in Eq. (4-5) have little explanatory power and the number of terms increases rapidly as the number of factors increases. Based on the same approach as that in Park (1992), the number of combinatorial product terms can be shown to be ($2^n - 1 - n$) where n is the number of factors in the decomposition analysis. As an example, the number of such product terms is 26 when n is equal to five, and five factors or more are now fairly common in IDA studies.

To address the problem, Sun (1998) introduces the principle of "jointly created and equally distributed" to distribute the combinatorial residual terms among the main effects. Using Sun’s approach and based on Eq. (4-2) to (4-4), we have

$$\Delta E_{act} = \sum_j (Y^T - Y^0) S_j^0 I_j^0 + \frac{1}{2} \sum_j (Y^T - Y^0)(I_j^T - I_j^0) S_j^0$$
$$+ \frac{1}{2} \sum_j (Y^T - Y^0)(S_j^T - S_j^0) I_j^0 + \frac{1}{3} \sum_j (Y^T - Y^0)(S_j^T - S_j^0)(I_j^T - I_j^0) \quad (4\text{-}6)$$

$$\Delta E_{str} = \sum_j Y^0 (S_j^T - S_j^0) I_j^0 + \frac{1}{2} \sum_j (Y^T - Y^0)(S_j^T - S_j^0) I_j^0$$
$$+ \frac{1}{2} \sum_j (I_j^T - I_j^0)(S_j^T - S_j^0) Y^0 + \frac{1}{3} \sum_j (Y^T - Y^0)(S_j^T - S_j^0)(I_j^T - I_j^0) \quad (4\text{-}7)$$

$$\Delta E_{int} = \sum_j Y^0 S_j^0 (I_j^T - I_j^0) + \frac{1}{2} \sum_j (Y^T - Y^0)(I_j^T - I_j^0) S_j^0$$
$$+ \frac{1}{2} \sum_j (I_j^T - I_j^0)(S_j^T - S_j^0) Y^0 + \frac{1}{3} \sum_j (Y^T - Y^0)(S_j^T - S_j^0)(I_j^T - I_j^0) \quad (4\text{-}8)$$

The Sun’s method has been adopted in a number of IDA studies. Sun and Ang (2000) extend the "jointly created and equally distributed" principle to the IDA methods formulated based on the Paasche index and M-E index. They found that with this extension all three methods give the same decomposition results.

4.2.2 Multiplicative Laspeyres-based IDA Methods

We study the general formula of multiplicative Laspeyres-based IDA methods, based on the general IDA formula for n factors and m sub-categories used in Section 2.3:

$$D_{x_i} = \frac{\sum_{j=1}^{m} [\frac{V^0}{x_i^0} + \alpha(\frac{V^T}{x_i^T} - \frac{V^0}{x_i^0})] \cdot x_i^T}{\sum_{j=1}^{m} [\frac{V^0}{x_i^0} + \alpha(\frac{V^T}{x_i^T} - \frac{V^0}{x_i^0})] \cdot x_i^0} \quad (4\text{-}9)$$

where $0 \leq \alpha \leq 1$. We have the multiplicative Laspeyres method when $\alpha = 0$, Paasche method when $\alpha = 1$, and the M-E method when $\alpha = 0.5$. These and other related IDA methods are referred to as "methods linked to the Laspeyres index" in the multiplicative form in Ang (2004).

Ang et al. (2004) extend the conventional two-factor Fisher index formula to the generalized Fisher index to complement the existing S/S IDA method in the additive approach. In Ang and Liu (2004), the generalized Fisher index is extended from the Shapley value method presented in Albrecht et al. (2002). According to Ang et al. (2004), the component for factor x_j ($j = 1, 2, \ldots, n$) using the Generalized Fisher index is given by

$$D_{x_i} = \prod_{\substack{S \subset N \\ i \in S}} \left[\frac{V(S)}{V(S \setminus \{i\})}\right]^{\frac{1}{n} \cdot \frac{1}{\binom{n-1}{s'-1}}} = \prod_{\substack{S \subset N \\ i \in S}} \left[\frac{V(S)}{V(S \setminus \{i\})}\right]^{\frac{(s'-1)!(n-s')!}{n!}} \tag{4-10}$$

$$V(S) = \sum \left(\prod\nolimits_{l \in S} x_l^T \prod\nolimits_{k \in N \setminus S} x_k^0\right) \tag{4-11}$$

$$V(\phi) = \sum \left(\prod\nolimits_{k \in N} x_k^0\right) \tag{4-12}$$

where $N = \{1, 2, \ldots, n\}$, the cardinality of N is n, S be a subset of N, the cardinality of S is s' and ϕ is a null subset.

4.3 Introduction of Shapley Value

The Shapley value was introduced by Shapley in 1953 as a solution to an n-person cooperative game. In this section, we give a short introduction of the Shapley value.

4.3.1 Cooperative Game Theory

Formally a game is defined on a finite set of players with a real-valued characteristic function which describes the worth of each coalition of the players. A solution of a game is simply a reward vector which gives the reward of each player. Game theory is divided into two branches; one is non-cooperative and the other is cooperative. Roca and Serrano (2007) define cooperative game theory as "it studies the interactions among coalitions of players. Its main question is that: Given the sets of feasible payoffs for each coalition, what payoff will be awarded to each player?"

4.3.2 Shapley Value in Cooperative Game Theory

In game theory, the Shapley value proposed by Shapley (1953) gives a fairly equitable solution to the cooperative game which can be considered in the characteristic function form or coalitional form (von Neumann and Morgenstern, 1944).

From Shapley (1953), the reward function or the Shapley value $\phi_i(v)$ of player i can be expressed as

$$\phi_i(v) = \sum_{\substack{R \subseteq N \\ i \in R}} \sum_{K \subseteq R} (-1)^{|R|-|K|} v(K) / |R| = \sum_{r=1}^{n} \sum_{\substack{R \subseteq N \\ i \in R \\ |R|=r}} \sum_{K \subseteq R} (-1)^{r-|K|} v(K) / r \qquad (4\text{-}13)$$

where $v(K)$ is the coalition value of players in set K. A simplified way of expressing the Shapley value (Shapley, 1953) is

$$\phi_i(v) = \sum_{i \in S \subseteq N, |S|=s} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S - \{i\})] \qquad (4\text{-}14)$$

From Eq. (4-14), we could find that the Shapely value quantifies the contribution of each player by calculating the marginal impact of each player when they are eliminated in sequence and averaging the estimated impacts for all possible elimination sequences to remove the influence of the order in which the plays appear in the elimination sequence. In Eq. (4-14), there are $\frac{n!}{(s-1)!(n-s)!}$ possible combinations (permutations) of the elimination sequences and $\frac{(s-1)!(n-s)!}{n!}$ is the probability of one elimination sequence of randomly selecting the subset S from N.

It has been proven that the Shapley value is the only reward vector that obeys three simple axioms (Shapley, 1953). The first or the symmetry axiom states that the reward vector is indifferent to the names of the players. The second or the carrier axiom is that a dummy player receives zero reward (null player axiom) and the sum of all rewards equals the coalition value of all players (efficiency axiom). The third or the additivity axiom says that the reward vector of the sum of two games is the sum of the individual reward vectors.

Besides game theory, the Shapley value has been widely applied in cost allocation and fair division. See, for example, Billera et al. (1978), Roth and Verrecchia (1979), and Moriarity (1983). In particular, a cost allocation problem can be placed in the same framework of a game and the Shapely value can be interpreted as a fair solution to the allocation of costs.

4.3.3 Shapley Value in IDA

IDA is a technique to study the impact of changes in a number of predefined factors of interest on energy-related aggregate. Shapley value is a solution of a game to evaluate the reward of each player given “the characteristic function specifying the value created by different subsets of the players in the game” (Brandenburger, 2007).

First, we study the similarities between IDA methods and solutions in cooperative game theory. The motivations of IDA methods and solutions in game theory are the same. IDA aims to study the effects of predefined factors on the change of an energy-related aggregate. Likewise, a solution in game theory is to study the reward of each player for the total gain of a cooperative game. It is in this spirit that the Shapley value was used by Albrecht et al. (2002) as a solution to the perfect decomposition of carbon emissions without residuals. In this context, a decomposition in IDA is equivalent to a solution in a game and a factor in IDA is a player in a game. The axioms in Shapley value may be comparable to some index number tests of the desirability of an IDA method. The symmetry and carrier axioms together are equivalent to the factor-reversal test, while the additive axiom has some similarity with the aggregation test.

Second, we will discuss the differences between IDA methods and solutions in cooperative game theory. In game theory, it is necessary to know all the "characteristic functions specifying the value created by different subsets of the players in the game". The condition to extend the Shapley value in IDA is that all the changes of an energy-related aggregate created by different subsets of the factors should be known to calculate the marginal impacts of factors. In IDA, the impacts of individual predefined factors are studied. When using Shapley value, both the effects of individual factors and the interactions among factors (residual in IDA studies) are considered. Take 3-factor IDA study as an example. Assuming total energy consumption from in Year 0 to in Year T is decomposed to give the impacts of overall industrial activity, activity structure and sector energy intensity. Using IDA methods, we would get the effects of activity, structure and energy intensity. When using the Shapley value, the energy consumption changes created by activity (structure; energy intensity) should be predefined. In addition, the energy consumption changes created by the interaction of activity and structure (interaction of activity and intensity; interaction of structure and intensity) should be given. Moreover, the energy consumption changes created by the interaction of activity, structure and energy intensity (this is the total energy consumption change between Year 0 and Year T) should be predefined.

4.4 Laspeyres-based IDA Methods and the Shapley Value

In this Section, we extend the Shapley value in IDA and show that not only the additive Laspeyres-based method, the multiplicative Laspeyres-based IDA methods are a form of Shapley values by identifying suitably defined characteristic functions.

4.4.1 Additive Laspeyres-based IDA Methods and the Shapley Value

In additive decomposition, changes in an energy-related aggregate are expressed in the form of the difference between the target year and the base year, which means that the additive measure analyzes the absolute change. In cooperative game theory, the total gain is shown in absolute change as well and the Shapley value can be adapted in additive IDA directly.

We identify the characteristic functions ν of Laspeyres-based methods in Table 4-1, where ν(S) gives the effects created by factors in S. In the table, we associate the various characteristic functions to the respective "index forms" instead of IDA methods so as to avoid any confusion that may arise.

Applying the characteristic function in the Laspeyres index form given in Table 4-1 in Eq. (4-13), the corresponding Shapley value for the i^{th} factor is

$$\phi_i(v)=\sum_{j=1}^{m}\sum_{r=1}^{n}\sum_{\substack{R\subseteq N\\ i\in R\\ |R|=r}}\sum_{K\subseteq R}(-1)^{r-|K|}v_j(K)/r=\sum_{j=1}^{m}\sum_{r=1}^{n}\sum_{\substack{R=\{l_1,l_2,\cdots,l_r\}\\ i=l_1}}\sum_{\mathrm{K}\subseteq\mathrm{R}}(-1)^{r-|K|}v_j(K)/r.$$

$$\sum_{\substack{R=\{l_1,l_2,\ldots,l_r\}\\ K\subseteq R}} (-1)^{r-|K|} v_j(K) = \sum_{\substack{R=\{l_1,l_2,\ldots,l_r\}\\ K\subseteq R}} (-1)^{r-|K|} \Big(\prod_{l\in K} x_{j,l}^T \prod_{p\in N\setminus K} x_{j,p}^0 - \prod_{i=1}^{n} x_{j,i}^0\Big)$$
$$= \sum_{\substack{R=\{l_1,l_2,\ldots,l_r\}\\ K\subseteq R}} (-1)^{r-|K|} \Big(\prod_{l\in K} x_{j,l}^T - \prod_{l\in K} x_{j,l}^0\Big) \prod_{p\in N\setminus K} x_{j,p}^0 \tag{4-15}$$

For each subset K of R,

$$\begin{aligned}\Big(\prod_{l\in K} x_{j,l}^T - \prod_{l\in K} x_{j,l}^0\Big) \prod_{p\in N\setminus K} x_{j,p}^0 &= \sum_{l\in K}\Big[(x_{j,l}^T - x_{j,l}^0) \prod_{p\in N\setminus\{l\}} x_{j,p}^0\Big] \\ &+ \sum_{l_1,l_2\in K}\Big[(x_{j,l_1}^T - x_{j,l_1}^0)(x_{j,l_2}^T - x_{j,l_2}^0) \prod_{p\in N\setminus\{l_1,l_2\}} x_{j,p}^0\Big] \\ &+\cdots \\ &+\prod_{l\in K}(x_{j,l}^T - x_{j,l}^0) \prod_{p\in N\setminus K} x_{j,p}^0 .\end{aligned}$$

We find its sum of coefficients is

$\sum_{k=1}^{r}(-1)^{r-k}\binom{r-1}{k-1} = \sum_{k=1}^{r}(-1)^{(r-1)-(k-1)}\binom{r-1}{k-1} = (1-1)^{r-1} = 0.$ Similarly for any fixed proper subset L of R, the sum of coefficients corresponding to the term

$\prod_{l\in L\subset R, L\neq R}(x_{j,l}^T - x_{j,l}^0) \prod_{p\in N\setminus L} x_{j,p}^0$ is

$\sum_{k=|L|}^{r}(-1)^{r-k}\binom{r-|L|}{k-|L|} = \sum_{k=|L|}^{r}(-1)^{(r-|L|)-(k-|L|)}\binom{r-|L|}{k-|L|} = (1-1)^{r-|L|} = 0.$ Now considering the terms corresponding to $\prod_{l\in R}(x_{j,l}^T - x_{j,l}^0) \prod_{p\in N\setminus R} x_{j,p}^0$, we find that there is only one of them and its coefficient is $(-1)^{r-r} = 1$. Hence,

$$\phi_i(v) = \sum_{j=1}^{m}\sum_{r=1}^{n} \sum_{\substack{R=\{l_1,l_2,\cdots,l_r\}\\ K\subseteq R}} (-1)^{r-|K|} v_j(K) = \prod_{l\in R}(x_{j,l}^T - x_{j,l}^0) \prod_{p\in N\setminus R} x_{j,p}^0$$
$$= \sum_{j=1}^{m}\sum_{r=1}^{n} \sum_{\substack{l_1,l_2,\cdots,l_r\\ i=l_1}} \frac{V^0}{x_{j,l_1}^0 x_{j,l_2}^0 \cdots x_{j,l_r}^0} \Delta x_{j,l_1}\Delta x_{j,l_2}\cdots\Delta x_{j,l_r} / r \tag{4-16}$$

where $\Delta x_{j,l_i} = x_{j,l_i}^T - x_{j,l_i}^0$, for $i = 1, 2, \cdots, r$.

The notion of a characteristic function in the Shapley value, which reveals the effect of any number of factors, has so far not been discussed in any IDA study. From Eq. (4-16), it is also shown that the same Shapley value solution to the three index forms matches the perfect decomposition introduced by Sun (1998) and Sun and Ang (2000).

In Appendix A, we prove that the Laspeyres, Paasche and M-E methods share the same Shapley value through some naturally defined characteristic functions. It is also shown in Appendix A that the same Shapley value solution to the three index forms matches the perfect decomposition introduced by Sun (1998) and Sun and Ang (2000). Thus it demonstrates that the approach introduced by Sun (1998) provides a fair and perfect decomposition solution to Laspeyres, Passche, and M-E methods with their residuals distributed in an equitable way.

It can be extended that for any value α in Eq. (4-1) a characteristic function may be naturally defined using the general formula in Table 4-1. Again it is shown in Appendix A that associated with this characteristic function its Shapley value coincides with the approach introduced by Sun (1998). Hence the Shapley value derived based on the respective characteristic functions provides a direct linkage to these IDA methods with the residual terms distributed according to the “jointly created and equally distributed” principle.

Table 4-1. Characteristic functions based on the additive Laspeyres, Paasche and M-E index forms and the general form

Laspeyres form	$v(S)=\sum_{j=1}^{m}(\prod_{l\in S} x_{j,l}^{T}\prod_{p\in N\backslash S} x_{j,p}^{0}-\prod_{i=1}^{n} x_{j,i}^{0})=\sum_{j=1}^{m} v_j(S)$ $v(N)=V^T-V^0$
Paasche form	$v(S)=-\sum_{j=1}^{m}(\prod_{l\in S} x_{j,l}^{0}\prod_{p\in N\backslash S} x_{j,p}^{T}-\prod_{i=1}^{n} x_{j,i}^{T})=\sum_{j=1}^{m} v_j(S)$ $v(N)=V^T-V^0$
M-E form	$v(S)=\sum_{j=1}^{m}[0.5*(\prod_{l\in S} x_{j,l}^{T}\prod_{p\in N\backslash S} x_{j,p}^{0}-\prod_{i=1}^{n} x_{j,i}^{0})$ $-0.5(\prod_{l\in S} x_{j,l}^{0}\prod_{p\in N\backslash S} x_{j,p}^{T}-\prod_{i=1}^{n} x_{j,i}^{T})]=\sum_{j=1}^{m} v_j(S)$ $v(N)=V^T-V^0$
General form for "indices linked to the Laspeyres index"	$v(S)=\sum_{j=1}^{m}[(1-\alpha)\cdot(\prod_{l\in S} x_{j,l}^{T}\prod_{p\in N\backslash S} x_{j,p}^{0}-\prod_{i=1}^{n} x_{j,i}^{0})$ $-\alpha\cdot(\prod_{l\in S} x_{j,l}^{0}\prod_{p\in N\backslash S} x_{j,p}^{T}-\prod_{i=1}^{n} x_{j,i}^{T})]=\sum_{j=1}^{m} v_j(S)$ $v(N)=V^T-V^0$

We use the data in Table 4-2 to present a simple numerical example. Direct application of the conventional Laspeyres, Paasche, M-E, and S/S methods give the results shown in Table 4-3. It can be seen that the residual term is fairly large for the first three methods. The results for the S/S method are obtained using Eq. (4-6) to (4-8) and the decomposition is perfect.

Table 4-4 lists the detailed characteristic function values of the Laspeyres, Paasche and M-E index forms derived directly from the formulae in Table 4-1 using the data in Table 4-2. In Table 4-4, x_1, x_2 and x_3 represent activity, structure and energy intensity variables, respectively. A comparison between

Table 4-3 and Table 4-4 shows that $v(\{x_1\})$, $v(\{x_2\})$ and $v(\{x_3\})$ give the estimates of ΔE_{act}, ΔE_{str} and ΔE_{int}, respectively, which are the main effects before the distribution of the residual terms using the conventional IDA methods.

Table 4-2. Data for a two-sector IDA example (arbitrary units)

	Year 0				Year T			
	E^0	Y^0	S^0	I^0	E^T	Y^T	S^T	I^T
Sector 1	30	10	0.2	3	80	40	0.5	2
Sector 2	20	40	0.8	0.5	16	40	0.5	0.4
Industry	50	50	1	1	96	80	1	1.2

Table 4-3. Decomposition results for the Laspeyres, Paasche, M-E and S/S methods based on the data in Table 4-2

IDA methods	**Laspeyres**	**Paasche**	**M-E**	**S/S**
ΔE_{tot}	46.00	46.00	46.00	46.00
ΔE_{act}	30.00	36.00	33.00	34.35
ΔE_{str}	37.50	38.40	37.95	39.30
ΔE_{int}	-14.00	-44.00	-29.00	-27.65
ΔE_{rsd}	-7.50	15.60	4.05	-

Table 4-4. Characteristic function values of the Laspeyres, Paasche, and M-E index forms obtained using the formulae in Table 4-1 and data in Table 4-2

Characteristic function	**Laspeyres form**	**Paasche form**	**M-E form**
$v(\{x_1\})$	30.00	36.00	33.00
$v(\{x_2\})$	37.50	38.40	37.95
$v(\{x_3\})$	-14.00	-44.00	-29.00
$v(\{x_1, x_2\})$	90.00	60.00	75.00
$v(\{x_1, x_3\})$	7.60	8.50	8.05
$v(\{x_2, x_3\})$	10.00	16.00	13.00
$v(\{x_1, x_2, x_3\})$	46.00	46.00	46.00

Following the allocation as defined in the Shapley value in Appendix A and taking the Laspeyres index case as an example, we are able to obtain the final decomposition results as follows, where the reward function or the Shapley value $\phi_i(v)$ of player i gives the IDA decomposition results contributed by the i^{th} factor:

For the activity effect (x_1):

$$\begin{aligned}\phi_{x_1}(v) &= \frac{2!}{3!}v(\{x_1\}) + \frac{1!\cdot 1!}{3!}[v(\{x_1, x_2\}) - v(\{x_2\})] \\ &+ \frac{1!\cdot 1!}{3!}[v(\{x_1, x_3\}) - v(\{x_3\})] + \frac{2!}{3!}[v(\{x_1, x_2, x_3\}) - v(\{x_2, x_3\})] \\ &= 34.35\end{aligned} \quad (4\text{-}17)$$

For the structure effect (x_2):

$$\begin{aligned}\phi_{x_2}(v) &= \frac{2!}{3!}v(\{x_2\}) + \frac{1!\cdot 1!}{3!}[v(\{x_1,x_2\}) - v(\{x_1\})] \\ &+ \frac{1!\cdot 1!}{3!}[v(\{x_2,x_3\}) - v(\{x_3\})] + \frac{2!}{3!}[v(\{x_1,x_2,x_3\}) - v(\{x_1,x_3\})] \\ &= 39.3\end{aligned} \quad (4\text{-}15)$$

For the intensity effect (x_3):

$$\begin{aligned}\phi_{x_3}(v) &= \frac{2!}{3!}v(\{x_3\}) + \frac{1!\cdot 1!}{3!}[v(\{x_1,x_3\}) - v(\{x_1\})] \\ &+ \frac{1!\cdot 1!}{3!}[v(\{x_2,x_3\}) - v(\{x_2\})] + \frac{2!}{3!}[v(\{x_1,x_2,x_3\}) - v(\{x_1,x_2\})] \\ &= -27.65\end{aligned} \quad (4\text{-}19)$$

The final results are exactly the same as those given by the S/S method in Table 4-3. Applying the same procedure to Paasche and M-E index cases, or in fact to any case that satisfies the general formula in Table 4-1, the same final decomposition results will be obtained, as proven in Appendix A.

4.4.2 Multiplicative Laspeyres-based IDA Methods and the Shapley Value

Ang and Liu (2004) fill a gap in IDA by extending the conventional two-factor Fisher index decomposition approach to n factors to complement an existing additive decomposition approach. In Ang and Liu (2004), the generalized Fisher index is extended from the S/S method presented in Albrecht et al. (2002). However, Ang and Liu (2004) lacks a formal treatment to the Shapley value in the IDA context. In this section, we shall formalize the relationships between methods linked to the multiplicative Laspeyres-based index and the Shapley value through defining the characteristic function in the Shapley value. We find that the Shapley value in multiplicative Laspeyres-based method is the same as the generalized Fisher index.

Since the Shapley value studies the absolute change, we could extend Shapley value in multiplicative IDA through the following transformation:

$$\ln D_{tot} = \ln \frac{V^T}{V^0} = \ln V^T - \ln V^0 = \sum_i \ln D_{x_i} \tag{4-20}$$

For additive IDA, the total change could be treated as the sum of individual sub-sectors. The Shapley value at an aggregate level could be treated as the sum of individual (sub-sector) Shapley value (additivity axiom). For multiplicative decomposition, due to the limitation of log-function, the energy-related aggregate problem is treated as a single game instead of the sum of individual games (sub-sector).

The characteristic functions for multiplicative Laspeyres method are

$$\begin{aligned} v(S) &= \ln(\sum_{j=1}^{m} \prod_{l \in S} x_{j,l}^{T} \prod_{p \in N \setminus S} x_{j,p}^{0}) - \ln(\sum_{j=1}^{m} \prod_{i=1}^{n} x_{j,i}^{0}) \\ v(N) &= \ln V^T - \ln V^0 \end{aligned} \tag{4-21}$$

According to Eq. (4-14)

$$\begin{aligned} \phi_{x_i} &= \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S - \{i\})] \\ &= \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} [\ln V(S) - \ln V(S - \{i\})] \\ &= \ln(\prod_{S \subseteq N} [V(S)/V(S-\{i\})]^{\frac{(s-1)!(n-s)!}{n!}}) = \ln(D_{x_i}^{Fisher-Modified}) \end{aligned} \tag{4-22}$$

where, $V(S)=\sum_{j=1}^{m}\prod_{l\in S}x_{j,l}^{T}\prod_{p\in N\backslash S}x_{j,p}^{0}$. Therefore, the generalized Fisher is a special case of the Shapley value method based on the multiplicative Laspeyres method.

The characteristic functions for multiplicative Paasche method are

$$\begin{aligned} &v(S)=\ln(\prod_{i=1}^{n}x_{j,i}^{T})-\ln(\sum_{j=1}^{m}(\prod_{l\in S}x_{j,l}^{0}\prod_{p\in N\backslash S}x_{j,p}^{T}) \\ &V(N)=\ln V^{T}-\ln V^{0} \end{aligned} \tag{4-23}$$

According to Eq. (4-14)

$$\begin{aligned} \phi_{x_i} &= \sum_{N\backslash(S-\{i\})\subseteq N}\frac{((n-s+1)-1)!(n-(n-s+1))!}{n!}[v(N\backslash(S-\{i\}))-v(N\backslash S)] \\ &= \sum_{N\backslash(S-\{i\})\subseteq N}\frac{(s-1)!(n-s)!}{n!}[\ln V(S)-\ln V(S-\{i\})] \qquad \text{4-24)} \\ &= \ln(\prod_{S\subseteq N}[V(S)/V(S-\{i\})]^{\frac{(s-1)!(n-s)!}{n!}})=\ln(D_{x_i}^{Fisher-Modified}) \end{aligned}$$

where, $V(S)=\sum_{j=1}^{m}\prod_{l\in S}x_{j,l}^{T}\prod_{p\in N\backslash S}x_{j,p}^{0}$. Therefore, the generalized Fisher is a special case of the Shapley value method based on the multiplicative Paasche method.

Similarly, we can see that M-E methods and the General form for multiplicative Laspeyres-based IDA methods share the same Shapley value, which is the generalized Fisher method through some naturally defined characteristic functions. These characteristic functions ν are displayed in Table 4-5, where ν(*S*) gives the effects created by factors in *S*. In the table, we associate the various characteristic functions to the respective "index forms" instead of IDA methods so as to avoid any confusion that may arise.

Table 4-5. Characteristic functions based on the multiplicative Laspeyres, Paasche and M-E index forms and the general form

Laspeyres form	$v(S)=\ln(\sum_{j=1}^{m}\prod_{l\in S}x_{j,l}^{T}\prod_{p\in N\backslash S}x_{j,p}^{0})-\ln(\sum_{j=1}^{m}\prod_{i=1}^{n}x_{j,i}^{0})$ $v(N)=\ln V^{T}-\ln V^{0}$
Paasche form	$v(S)=\ln(\prod_{i=1}^{n}x_{j,i}^{T})-\ln(\sum_{j=1}^{m}(\prod_{l\in S}x_{j,l}^{0}\prod_{p\in N\backslash S}x_{j,p}^{T})$ $V(N)=\ln V^{T}-\ln V^{0}$
M-E form	$v(S)=\ln[\sum_{j=1}^{m}0.5*(\prod_{i=1}^{n}x_{j,i}^{T}+\prod_{l\in S}x_{j,l}^{T}\prod_{p\in N\backslash S}x_{j,p}^{0})]$ $-\ln[\sum_{j=1}^{m}0.5*(\prod_{l\in S}x_{j,l}^{0}\prod_{p\in N\backslash S}x_{j,p}^{T}+\prod_{i=1}^{n}x_{j,i}^{0})]$ $v(N)=\ln V^{T}-\ln V^{0}$
General form for "indices linked to the Laspeyres index"	$v(S)=\ln[\sum_{j=1}^{m}(\alpha\prod_{i=1}^{n}x_{j,i}^{T}+(1-\alpha)\cdot\prod_{l\in S}x_{j,l}^{T}\prod_{p\in N\backslash S}x_{j,p}^{0})]$ $-\ln[\sum_{j=1}^{m}(\alpha\cdot\prod_{l\in S}x_{j,l}^{0}\prod_{p\in N\backslash S}x_{j,p}^{T}-(1-\alpha)\prod_{i=1}^{n}x_{j,i}^{0})]$ $v(N)=\ln V^{T}-\ln V^{0}$

4.5 Conclusion

We have formalized the relationships between the Laspeyres-based methods, and the Shapley value. It is shown that the linkage can be established through defining the characteristic function in the Shapley value, and the "jointly created and equally distributed" principle proposed in Sun (1998) is equivalent to the allocation principle in the Shapley value. Following this line of reasoning, the principle of the Shapley value can be further extended to cover some other IDA methods in a unified and coherent manner.

CHAPTER 5: Divisia-based Index Decomposition Analysis Methods

5.1 Introduction

The Divisia index was first introduced to IDA in Boyd et al. (1987). Since then, various Divisia-based IDA methods have been developed, and some of them have become popular in research work and application studies. AMDI is derived from the Divisia index in Boyd et al. (1988). Liu et al. (1992) transformed the Divisia integral path problem into a parameter estimation problem and proposed two groups of general parametric Divisia methods, PDM1 and PDM2. Ang and Choi (1997) propose a refined Divisia index method, the multiplicative LMDI II method, based on the study of Sato (1976) in INP. Ang et al. (1998) introduce the additive LMDI I method based on studies by Sato (1976) and Tornqvist et al. (1985). Ang and Liu (2001) present the multiplicative LMDI I in IDA and introduce the concept of consistency in aggregation in the energy decomposition context for the first time.

Ang (2004) points out a simple relationship between the additive and multiplicative forms for both LMDI I and LMDI II, and recommends LMDI I method as the preferred IDA method. In this section, we further show that there exists a simple and meaningful relationship among most of the methods linked to the Divisia index, including that between AMDI and LMDI I. With these findings, we are able to extend the findings in Ang (2004). This helps to improve our understanding of the properties of popular IDA methods and IDA methodology in general. The findings are also useful to analysts in method selection and decomposition result interpretation. After a discussion about the properties and linkages among the Divisia-based methods, we study properties of Divisia-based IDA methods and attempt to provide recommendations in method selection.

The next section gives an introduction to additive Divisia-based IDA methods including the general formulae. It is followed by discussions on multiplicative Divisia-based methods in Section 5.3. Comparison among Divisia-based IDA methods is discussed in Section 5.4. The conclusion and recommendations are presented in Section 5.5.

5.2 Additive Divisia-based IDA Methods

In this section, we study the properties and linkages of popular Divisia-based IDA methods applied to decomposition of changes of an aggregate that take the additive form.

5.2.1 Formulae of Additive Divisia-based IDA Methods

The formulae of additive Divisia-based IDA methods may be studied based on the general formulae of IDA presented in Section 2.3. To study how an aggregate is affected by changes in the factors on the right hand side of Eq. (2-1) using this index, we take the differentiation of Eq. (2-1) with respect to time:

$$dV = \sum_j dV_j = \sum_j V_j (d \ln x_{j,1} + d \ln x_{j,2} + \cdots + d \ln x_{j,n}) \tag{5-1}$$

Integrating from 0 to *T*,

$$\Delta V = \int_0^T \sum_j dV_j = \sum_j \left(\int_0^T V_j d\ln x_{j,1} + \int_0^T V_j d\ln x_{j,2} + \cdots + \int_0^T V_j d\ln x_{j,n}\right) \quad (5\text{-}2)$$

In empirical studies, data are available only at the end points and Eq. (5-2) may be approximated by

$$\Delta V \cong \sum_j w_j \cdot \left(\int_0^T d\ln x_{j,1} + \int_0^T d\ln x_{j,2} + \cdots + \int_0^T d\ln x_{j,n}\right)$$

$$\Delta V \cong \sum_j w_j \cdot \ln\left(\frac{x_{j,1}^T}{x_{j,1}^0}\right) + \sum_j w_j \cdot \ln\left(\frac{x_{j,2}^T}{x_{j,2}^0}\right) + \cdots + \sum_j w_j \cdot \ln\left(\frac{x_{j,n}^T}{x_{j,n}^0}\right) \quad (5\text{-}3)$$

where w_j is a weight function. From Eq. (2-2) and Eq. (5-3), the effect associated with factor *i* is given by

$$\Delta V_{x_i} = \sum_{j=1}^m w_j \cdot \ln\left(\frac{x_{j,i}^T}{x_{j,i}^0}\right) \quad (5\text{-}4)$$

and *m* is the number of sub-categories in the data.

In the literature, the various IDA methods linked to the Divisia index are differentiated primarily by the weight function. We consider four such methods: AMDI, LAS-PDM1, LMDI I and LMDI II. From Eq. (5-4), their weight functions are respectively given by

$$w_j = \frac{1}{2}(V_j^T + V_j^0) \quad (5\text{-}5)$$

$$w_j = V_j^0 \quad (5\text{-}6)$$

$$w_j = L(V_j^T, V_j^0) \quad (5\text{-}7)$$

$$w_j = \frac{L(V_j^T / V^T, V_j^0 / V^0)}{\sum_{j=1}^{m} L(V_j^T / V^T, V_j^0 / V^0)} L(V^T, V^0) \tag{5-8}$$

The AMDI is proposed in Boyd et al. (1988), LAS-PDM1 in Ang and Lee (1994), LMDI I in Ang et al. (1998) and LMDI II in Ang et al. (2003).

Of these four methods, AMDI, LMDI I and LMDI II have been widely used to decompose changes in national and sectoral energy consumption and energy-related carbon emissions. A main difference among these four methods is that LMDI I and LMDI II give perfect decomposition results while LAS-PDM1 and AMDI do not. In view of its many desirable properties, LMDI I has been popular among researchers and analysts. It has been adopted by Canada (Natural Resources Canada, 2006) and Australia (Sandu and Syed, 2008) to study aggregate energy efficiency/intensity trends.

5.2.2 LMDI I as a General Form of Additive Divisia-based Methods

Comparisons are made among AMDI, LMDI I and LMDI II in Ang (2004) but no formal relationships are specified. We have found that there exists a simple relationship between those methods that take the form of Eq. (5-3). For these methods, a residual term occurs at each sub-category *j* in general. The sum of these residual terms over all sub-categories in the data set gives the overall residual term of the decomposition. More specifically, if the residual term of each sub-category is proportionately distributed according to the effects estimated for that sub-category and if this is done for all sub-categories, the final decomposition results will be exactly the same as those for LMDI I.

We shall refer to the distribution of the residual terms in this manner as the principle of "proportionally distributed by sub-category". For the "convergence" property that leads to LMDI I after applying this principle, the only condition is that the weight functions w_j in Eq. (5-4) should have the same mathematical form for different factors in the same sub-category. Both AMDI and LAS-PDM1 satisfy this condition and so do LMDI I and LMDI II. A numerical example is given in Section 5.2.4. The relationship that we have found is interesting because it provides a formal and yet very simple relationship between various IDA methods linked to the Divisia index and LMDI I. In particular, it relates AMDI to LMDI I, two of the most often used IDA methods in the literature. As a result of this finding, the attractiveness of the LMDI I in IDA is strengthened from the methodological viewpoint.

5.2.3 Relationship between Additive LMDI II and LMDI I

Both LMDI II and LMDI I give decomposition results which are perfect with no residual term, but the estimates of each of the effects given by the two methods tend to differ slightly. Earlier studies have compared the two methods and from an index number property viewpoint, both have their strengths and weaknesses. Both methods satisfy most of the tests of index numbers which are considered to be relevant to IDA, except that additive LMDI I fails the proportionality test while the additive LMDI II fails the aggregation test. It is based primarily on ease of use that between the two and Ang (2004) recommends LMDI I. While the additive LMDI I has been chosen by the national energy agencies of Canada and Australia, it is interesting that the

Office of Energy Efficiency and Renewable Energy of the United States (EERE, 2003) has opted for the multiplicative LMDI II method.

We have found that a difference between LMDI I and LMDI II is the residual terms at the sub-category level. LMDI I is perfect in decomposition at the sub-category level but this is not the case for LMDI II. This also means that, although LMDI II does not give decomposition results which are perfect at the sub-category level, the sum of the residual terms for all the sub-categories is always zero so that overall the method still gives results which are perfect in decomposition.

Furthermore, since the weight function of LMDI II takes the same form for all factors in the same sub-category, we can treat it just like AMDI or LAS-PDM1 and apply the principle of "proportionally distributed by sub-category" to the sub-category residual terms. The final decomposition results are exactly the same as those of LMDI I. The proof is a special case of the general proof and is shown below.

Starting from Eq. (5-4), we distribute the residual term of each sub-category proportionately according to the effects estimated for that sub-category, i.e., by applying the principle of the "proportionally distributed by sub-category", the resulting estimate of the contributions from factor x_i is given by:

$$\Delta V_{x_i} = \sum_{j=1}^{m} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0}) + \sum_{j=1}^{m} \{[(V_j^T - V_j^0) - \sum_{i=1}^{n} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})] \frac{w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}{\sum_{i=1}^{n} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}\}$$

$$= \sum_{j=1}^{m} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0}) + \sum_{j=1}^{m} \frac{(V_j^T - V_j^0) \cdot w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}{\sum_{i=1}^{n} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})} - \sum_{j=1}^{m} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})$$

$$= \sum_{j=1}^{m} \frac{(V_j^T - V_j^0) \cdot w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}{\sum_{i=1}^{n} w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}$$

$$= \sum_{j=1}^{m} \frac{(V_j^T - V_j^0) \cdot w_j \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}{w_j \cdot \ln(\frac{V_j^T}{V_j^0})}$$

$$= \sum_{j=1}^{m} \frac{(V_j^T - V_j^0) \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})}{\ln(\frac{V_j^T}{V_j^0})} = \sum_{j=1}^{m} L(V_j^T, V_j^0) \cdot \ln(\frac{x_{j,i}^T}{x_{j,i}^0})$$

The resulting estimate is exactly equal to that given by LMDI I.

5.2.4 A Numerical Example

The findings in Section 5.2.2 and 5.2.3 can be illustrated using a simple example on industrial energy decomposition. Industry comprises only two sectors and the data are shown in Table 5-1. Following the standard IDA formulation on industry energy consumption analysis, the increase in total energy consumption (E) from 50 units in Year 0 to 96 units in Year T is decomposed to give the impacts of overall industrial activity (Y), activity structure ($S_j = Y_j/Y$) and sector energy intensity ($I_j = E_j/Y_j$):

$$E = \sum_j E_j = \sum_j Y \cdot \frac{Y_j}{Y} \cdot \frac{E_j}{Y_j} = \sum_j Y \cdot S_j \cdot I_j \qquad (5\text{-}9)$$

and

$$\Delta E_{tot} = \Delta E_{act} + \Delta E_{str} + \Delta E_{int} + \Delta E_{rsd} \qquad (5\text{-}10)$$

Subscript *j* denotes sub-category ("industrial sector" in this example), *act*, *str* and *int* refer to the effects associated with the overall activity level, activity structure and sector energy intensity respectively, and *rsd* denotes the residual term.

Table 5-1. Data for a two-sector IDA example (arbitrary units)

	Year 0				Year *T*			
	E^0	Y^0	S^0	I^0	E^T	Y^T	S^T	I^T
Sector 1	30	10	0.2	3	80	40	0.5	2
Sector 2	20	40	0.8	0.5	16	40	0.5	0.4
Industry	50	50	1	1	96	80	1	1.2

From Eq (5-4), estimates of the effects are given by

$$\Delta E_{act} = \sum_{j=1}^{m} w_j \cdot \ln\left(\frac{Y^T}{Y^0}\right) \qquad (5\text{-}11)$$

$$\Delta E_{str} = \sum_{j=1}^{m} w_j \cdot \ln\left(\frac{S_j^T}{S_j^0}\right) \qquad (5\text{-}12)$$

$$\Delta E_{int} = \sum_{j=1}^{m} w_j \cdot \ln\left(\frac{I_j^T}{I_j^0}\right) \qquad (5\text{-}13)$$

where m = 2. The weight functions for AMDI, LAS-PDM1, LMDI I and LMDI II are respectively

$$w_j = \frac{1}{2}(E_j^T + E_j^0) \tag{5-14}$$

$$w_j = E_j^0 \tag{5-15}$$

$$w_j = L(E_j^T, E_j^0) \tag{5-16}$$

$$w_j = \frac{L(E_j^T / E^T, E_j^0 / E^0)}{\sum_{j=1}^{m} L(E_j^T / E^T, E_j^0 / E^0)} L(E^T, E^0) \tag{5-17}$$

We use AMDI is taken as an example to illustrate the application of the principle of the “proportionally distributed by sub-category” to the residual terms. The data are shown in Table 5-2.

Table 5-2. Decomposition results for AMDI before and after the application of the principle of the “proportionally distributed by sub-category” to the residual terms for the data in Table 5-1

	Sector 1			Sector 2			Industry		
	Original estimate	Distributed residual	Adjusted estimate	Original estimate	Distributed residual	Adjusted estimate	Original estimate	Adjusted estimate	LMDI I
ΔE_{tot}	50	-	50	-4	-	-4	46	46	46
ΔE_{act}	25.85	-1.89	23.96	8.46	-0.03	8.43	34.31	32.38	32.38
ΔE_{str}	50.40	-3.69	46.71	-8.46	0.03	-8.43	41.94	38.28	38.28
ΔE_{int}	-22.30	1.63	-20.67	-4.02	0.02	-4	-26.32	-24.67	-24.67
ΔE_{rsd}	-3.95	-	-	0.02	-	-	-3.93	-	-

As an example, the residual for sector j is given by

$$(E_j^T - E_j^0) - \frac{1}{2}(E_j^T + E_j^0) \cdot \ln(\frac{E_j^T}{E_j^0}) \tag{5-18}$$

which is -3.95 for Sector 1. Distributing this residual proportionally according to the principle, in this case according to the estimates of the three main effects given in the same column in the table, leads to the results shown in the column “distributed residual” and then the "adjusted estimate". The overall estimates of the effects before and after the distribution of the residual terms of the two sectors are given in the third last and second last columns of the table. For comparisons, the last column gives the actual estimates given by LMDI I. Table 5-3 and Table 5-4 show the results for LAS-PDM1 and LMDI II respectively. It is interesting to note from Table 5-4 that in the case of LMDI II, the individual sub-category residual terms are non-zero but their sum is equal to zero.

Table 5-3. Decomposition results for LAS-PDM1 before and after the application of the principle of the “proportionally distributed by sub-category” to the residual terms for the data in Table 5-1.

	Sector 1			Sector 2			Industry		
	Original estimate	Distributed residual	Adjusted estimate	Original estimate	Distributed residual	Adjusted estimate	Original estimate	Adjusted estimate	LMDI I
ΔE_{tot}	50	-	50	-4	-	-4	46	46	46
ΔE_{act}	14.10	9.86	23.96	9.40	-0.97	8.43	23.5	32.38	32.38
ΔE_{str}	27.49	19.22	46.71	-9.40	0.97	-8.43	18.09	38.28	38.28
ΔE_{int}	-12.16	-8.51	-20.67	-4.46	0.46	-4.00	-16.63	-24.67	-24.67
ΔE_{rsd}	20.58	-	-	0.46	-	-	21.04	-	-

Table 5-4. Decomposition results for LMDI II before and after the application of the principle of the "proportionally distributed by sub-category" to the residual terms for the data in Table 5-1

	Sector 1			Sector 2			Industry		
	Original estimate	Distributed residual	Adjusted estimate	Original estimate	Distributed residual	Adjusted estimate	Original estimate	Adjusted estimate	LMDI I
ΔE_{tot}	50	-	50	-4	-	-4	46	46	46
ΔE_{act}	24.10	-0.14	23.96	9.04	-0.62	8.43	33.14	32.38	32.38
ΔE_{str}	46.98	-0.27	46.71	-9.04	0.62	-8.43	37.94	38.28	38.28
ΔE_{int}	-20.79	0.12	-20.67	-4.29	0.29	-4.00	-25.08	-24.67	-24.67
ΔE_{rsd}	-0.29	-	-	0.29	-	-	-	-	-

5.2.5 Handling Zero Values in AMDI

A potential problem in the application of IDA methods linked to the Divisia index is how to handle zero values if they appear in the data. Zero values tend to occur when IDA is applied to decompose changes in energy-related carbon emissions. Ang and Choi (1997) and Ang and Liu (2007) show that when these values are replaced by a small number, LMDI I and LMDI II give converging results. AMDI does not possess this property and therefore has implementation problems.

It can now be shown that when the zero values are replaced by a small number, AMDI gives some extremely large residual terms for the affected sub-categories. It can also be shown that no matter how large these residual terms are, if the "proportionally distributed by sub-category" principle is followed, they will be distributed accordingly and the final decomposition results are exactly the same as those for LMDI I. Hence AMDI can in principle be applied if there are zero values in the data although it seems obvious that one would rather opt for LMDI I in such situations.

5.3 Multiplicative Divisia-based IDA Methods

In this Section, we study the multiplicative Divisia-based IDA methods. First, formulae of multiplicative Divisia-based IDA methods are provided. Next, it is shown that LMDI I is the only perfect method which satisfies consistent in aggregation property in multiplicative decomposition based on its formula.

5.3.1 Formulae of Multiplicative Divisia-based IDA Methods

To study how an aggregate is affected by changes in the factors on the right hand side of Eq. (2-1) using this index, we apply the theorem of instantaneous growth rate to Eq. (2-1) and this leads to:

$$d\ln(V)/dt = \sum_j \frac{V_j}{V}[d(\ln x_{j,1})/dt + d(\ln x_{j,2})/dt + \cdots + d(\ln x_{j,n})/dt] \quad (5\text{-}19)$$

Integrating from 0 to T,

$$\ln(\frac{V^T}{V^0}) = \sum_j [\int_0^T \frac{V_j}{V} d(\ln x_{j,1})/dt + \int_0^T \frac{V_j}{V} d(\ln x_{j,2})/dt + \cdots + \int_0^T \frac{V_j}{V} d(\ln x_{j,n})/dt] \quad (5\text{-}20)$$

Exponentiating Eq. (5-20) can be expressed in the multiplicative form: $D_{tot}^{0,T} = V^T / V^0 = D_1^{0,T} \cdot D_2^{0,T} \cdot \ldots \cdot D_n^{0,T} \cdot D_{rsd}^{0,T}$

where,

$$D_{x_i} = \exp\left\{ \int_0^T \sum_{j=1}^m \frac{V_j}{V} d\ln(x_{j,i})/dt \right\} \quad (5\text{-}21)$$

In empirical studies data are available only at the end points and Eq. (5-21) can be approximated by

$$D_{x_i} = \exp\left\{\sum_{j=1}^{m} w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}\right\} \tag{5-22}$$

where w_j is a weight function and m is the number of sub-categories in the data.

The various multiplicative IDA methods linked to the Divisia index are differentiated primarily by the weight function. Here, four such methods are considered: AMDI, LAS-PDM1, LMDI I and LMDI II. From Eq. (5-22), their weight functions are respectively given by

$$w_j = \frac{1}{2}\left(\frac{V_j^T}{V^T} + \frac{V_j^0}{V^0}\right) \tag{5-23}$$

$$w_j = \frac{V_j^0}{V^0} \tag{5-24}$$

$$w_j = \frac{L(V_j^T, V_j^0)}{L(V^T, V^0)} \tag{5-25}$$

$$w_j = \frac{L(V_j^T / V^T, V_j^0 / V^0)}{\sum_{j=1}^{m} L(V_j^T / V^T, V_j^0 / V^0)} \tag{5-26}$$

5.3.2 Consistency in Aggregation in Multiplicative Decomposition

Ang and Liu (2001) is the first to introduce the multiplicative LMDI I method and the property of consistency in aggregation in the energy decomposition analysis. Ang and Liu (2001) present that consistency in aggregation property is derived from INP given in Vartia (1976) and Diewert (1978) and prove that the multiplicative LMDI I method is consistent in aggregation. In this section, we extend the findings of Ang and Liu (2001) and prove that the LMDI I method is the only perfect IDA method for consistency in aggregation property in multiplicative decomposition.

In economy-wide energy efficiency accounting frameworks, an economy is usually divided into several major sectors, namely transport, industrial, residential and commercial/service. Each sector may be further divided into various sub-sectors. When multi-level disaggregation exists, IDA can be studied using both one-step and multi-step analysis. It is desirable that the decomposition results of one-step analysis and of multi-step analysis are consistent. Otherwise, the IDA study will be framework-based, which will reduce its explanatory power. This property is called consistency in aggregation.

The definition of consistency in aggregation in economics is described in Section 3.4. According to Vartia (1976), an index is said to be consistent in aggregation when the index for an aggregate has the same value no matter whether it is calculated directly in a single operation without distinguishing the sub-index for each sub-aggregate or in two or more steps by first calculating separate indexes for its sub-aggregates and then aggregating them. In addition, the formulae for both frameworks and each step within a framework should be the same.

Using the general formulae studied in Section 2.3, for sub-sector j, the ratio change of energy-related aggregate is given as

$$D_j = V_j^T / V_j^0 = D_{j,1} \cdot D_{j,2} \cdot \ldots \cdot D_{j,n} \cdot D_{j,rsd} \tag{5-27}$$

D_j in Eq. (5-27) could be rewritten as:

$$D_j = \frac{V_j^T}{V_j^0} = \exp\left[\frac{V_j^T - V_j^0}{L(V_j^T, V_j^0)}\right] \tag{5-28}$$

At a higher aggregation, the ratio change of energy-related aggregate is given as

$$D_{tot} = V^T / V^0 = D_1 \cdot D_2 \cdot \ldots \cdot D_n \cdot D_{rsd} \tag{5-29}$$

D_{tot} in Eq. (5-29) could be rewritten as:

$$D_{tot} = \frac{V^T}{V^0} = \exp\left[\frac{V^T - V^0}{L(V^T, V^0)}\right] \tag{5-30}$$

Since $V^T - V^0 = \sum_j (V_j^T - V_j^0)$

$$D_{tot} = \frac{V^T}{V^0} = \exp\left[\frac{V^T - V^0}{L(V^T, V^0)}\right] = \exp\left[\frac{\sum_j (V_j^T - V_j^0)}{L(V^T, V^0)}\right] \tag{5-31}$$

From Eq. (5-28), we could get

$$V_j^T - V_j^0 = \ln D_j \cdot L(V_j^T, V_j^0) \tag{5-32}$$

Using Eq. (5-32), Eq.(5-31) could be rewritten as:

$$
\begin{aligned}
D_{tot} &= \frac{V^T}{V^0} = \exp\left[\frac{V^T\text{-}V^0}{L(V^T,V^0)}\right] = \exp\left[\frac{\sum_j (\ln D_j \cdot L(V_j^T,V_j^0))}{L(V^T,V^0)}\right] \\
&= \exp\left[\sum_j \frac{L(V_j^T,V_j^0)}{L(V^T,V^0)} \cdot \ln D_j\right] \\
&= \exp\left[\sum_j \sum_i \frac{L(V_j^T,V_j^0)}{L(V^T,V^0)} \cdot \ln D_{ij}\right] \\
&= \prod_i \exp\left[\sum_j \frac{L(V_j^T,V_j^0)}{L(V^T,V^0)} \cdot \ln D_{ij}\right]
\end{aligned}
\tag{5-33}
$$

From Eq. (5-33), we find that the aggregation of sub-sectors ratio indices to a higher level ratio index is based on the LMDI I formula. Consistency in aggregation property means that the aggregation function and the decomposition function have the same formula, and the final result is consistent. From the study above, we can see that multiplicative LMDI I method meets this condition and is the only perfect method that satisfies consistency in aggregation in the multiplicative form.

5.3.3 Empirical Study

In this section, we illustrate the property of consistency in aggregation using an empirical study of the residential sector of the United States economy from 1990 to 2002 (EERE, 2009) to show the advantages of IDA methods having consistency in aggregation. The residential sector is disaggregated into four areas: Northeast, Mideast, South and West. Each area is further disaggregated into five types of households: Single-Family Detached, Single-Family Attached, Mobile Home, Multi-Family (2-4 units) and Multi-Family (>4 units). Two levels of disaggregation are considered as shown in Table 5-5.

Table 5-5. Data on energy consumption and activity for residential sector in US economy, 1990 and 2002

Level 0	Level 1	Level 2	1990			2002		
			Energy (TBtu)	Activity (Million HH)	Intensity (TBtu/ Million HH)	Energy (TBtu)	Activity (Million HH)	Intensity (TBtu/ Million HH)
Residential	Northeast	Single-Family Detached	2093.0	9.73	215.11	1979.5	9.39	210.81
		Single-Family Attached	339.8	2.09	162.58	455.5	2.70	168.70
		Mobile Home	77.4	0.50	154.80	104.8	0.71	147.61
		Multi-Family (2-4 units)	520.1	3.48	149.45	431.1	3.1	139.06
		Multi-Family (>4 units)	322.9	3.28	98.45	409.5	4.45	92.02
	Midwest	Single-Family Detached	3241.5	15.19	213.40	3828.8	16.66	229.82
		Single-Family Attached	124.8	0.70	178.29	320.2	1.81	176.91
		Mobile Home	258.2	1.59	162.39	222.2	1.21	183.64
		Multi-Family (2-4 units)	401.6	2.48	161.94	370.9	2.22	167.07
		Multi-Family (>4 units)	282.7	2.98	94.87	267.3	2.83	94.45
	South	Single-Family Detached	4377.4	21.65	202.19	5793.8	25.93	223.44
		Single-Family Attached	286.4	1.89	151.53	506.4	2.93	172.83
		Mobile Home	274.9	1.89	145.45	624.1	3.33	187.42
		Multi-Family (2-4 units)	301.3	2.38	126.60	356.6	2.52	141.51
		Multi-Family (>4 units)	465.3	4.27	108.97	517.5	4.55	113.74
	West	Single-Family Detached	1892.7	11.42	165.74	2176.2	12.63	172.30
		Single-Family Attached	201.4	1.19	169.24	266.6	2.30	115.91
		Mobile Home	163.9	1.19	137.73	277.0	1.71	161.99
		Multi-Family (2-4 units)	176.0	1.69	104.14	148.2	1.61	92.05
		Multi-Family (>4 units)	302.6	3.77	80.27	408.1	5.36	76.14

TBtu, Trillion British Thermal Units; Million HH, Million Households.

We decompose energy consumption ($E = \sum_j E_j = \sum_j A_j \cdot \frac{E_j}{A_j}$) into two factors (sectoral activity and sectoral energy intensity effects) using the popular Divisia-based IDA methods: LMDI I, LMDI II and AMDI. Table 5-6 shows the results of sub-residential in this case study. In two-step analysis, the decomposition results are first calculated from the household type level to the household area level as shown in Table 5-6. The decomposition results in

Table 5-6 are then aggregated to the residential level. In one-step analysis, the decomposition results are obtained from household type level directly. The results for one-step analysis and two-step analysis are summarized in Table 5-7. We can find that both LMDI I and LMDI II provide perfect decomposition with no residual, and AMDI has residual terms. The comparison of one-step and two-step analysis in Table 5-7 shows that LMDI I gives consistent results for both one-step and two-step analysis, while the results for LMDI II and AMDI are not consistent. As a result, our study suggests that LMDI I is preferred in performing multi-step analysis in multiplicative decomposition.

Table 5-6. Results of sub-residential study (multiplicative decomposition)

Sub-sector	Effect	LMDI I	LMDI II	AMDI
Northeast	Activity	1.03529	1.03535	1.03564
	Intensity	0.97374	0.97368	0.97369
	Residual	1	1	0.99972
Mideast	Activity	1.08991	1.08966	1.09154
	Intensity	1.06667	1.06692	1.06674
	Residual	1	1	0.99846
South	Activity	1.22732	1.22724	1.22787
	Intensity	1.11370	1.11377	1.11398
	Residual	1	1	0.99931
West	Activity	1.20221	1.20225	1.20230
	Intensity	0.99575	0.99572	0.99574
	Residual	1	1	0.99997

Table 5-7. Results of residential study (multiplicative decomposition) for both one-step and two-step aggregation

Residential (one-step)	LMDI I	LMDI II	AMDI
Activity	1.14761	1.14755	1.14823
Intensity	1.05320	1.05325	1.05332
Residual	1	1	0.99935
Residential (two-step)	LMDI I	LMDI II	AMDI
Activity	1.14761	1.14756	1.14834
Intensity	1.05320	1.05324	1.05326
Residual	1	1	0.99931

5.4 Method Recommendation

In the study above, a simple linkage among popular Divisia-based IDA methods is found in additive. After applying the "proportionally distributed by sub-category" principle to the residual terms, both AMDI and LMDI II have the same results with LMDI I, which supports the conclusion that LMDI I is superior in additive Divisia-based IDA methods.

In Chapter 3, we summarized various important tests in IDA. Ang (2004) shows that LMDI I, LMDI II and AMDI all satisfy the time reversal test. However, AMDI does not satisfy factor reversal test, since the decomposition results of AMDI have residual terms. Ang (2004) points out that there is a simple and meaningful relationship between LMDI I and LMDI II in the additive and multiplicative forms. There is no connection between additive and multiplicative AMDI decomposition analysis. In addition to the zero value problems, in AMDI, LMDI I and II are preferred IDA methods. In Section 5.3, we report results which suggest that LMDI I methods are the only perfect method owning consistency in aggregation property in multiplicative decomposition, which shows that LMDI I is superior to LMDI II in multiplicative decomposition.

In summary, the findings of our study suggest that LMDI I is the preferred Divisia-based IDA method from both the theoretical and application viewpoints.

5.5 Conclusion

In the additive form, we found that most Divisia-based IDA methods, including AMDI and LMDI II, collapse to LMDI I after applying the "proportionally distributed by sub-category" principle to the residual terms. The principle provides a formal linkage between AMDI and LMDI I which have previously been considered to be unrelated. Furthermore, with this linkage, the problem that AMDI fails when there are zero values in the data, as was seen in the past, can now be resolved. For LMDI I and LMDI II, Ang (2004) treats LMDI I more favorably, mainly on the basis of ease of application, since it has a simpler weight function and is therefore easier to apply. Our study shows that application of the "proportionally distributed by sub-category" principle to LMDI II leads to LMDI I.

In multiplicative form, we found that the multiplicative LMDI I method is the only perfect method that satisfies consistency in aggregation property. In addition, we compared the popular Divisia-based IDA methods and, based on our findings, recommend LMDI I as the preferred IDA method from both the theoretical and application viewpoints.

CHAPTER 6: Chaining versus Non-chaining Approach

6.1 Introduction

Boyd et al. (1987) use yearly time-series data to study US industrial energy consumption and the chaining approach was adopted. It is the first time that chaining approach was used in IDA studies. Since then, both chaining and non-chaining approaches have been used in IDA studies to track energy efficiency trends. Some researchers prefer the chaining approach and examples of such studies include Liu et al. (2007) and Choi and Ang (2012). There are also studies that use non-chaining approach, for instance, Shrestha et al. (2007). It is also noted that there are studies that use both approaches (Hatzigeorgiou et al., 2008).

From the literature review in Chapter 2, it is clear that both the chaining and non-chaining approaches are as widely used in recent years. In addition, from the review, we find that the terminologies for chaining and non-chaining are not consistent in different IDA studies. They include “time series (i.e. yearly) decomposition” and “period-wise decomposition” (Ang and Lee, 1994 and Liu et al., 2007), “rolling base year decomposition” and “fixed base year decomposition” (Greening et al., 1997 and Bataille and Nyboer, 2005), and “chaining decomposition” and “non-chaining decomposition” (Ang, 2004 and Ang and Liu, 2007a). In our study, we opt to use “chaining decomposition” and “non-chaining decomposition”, so that the terminologies are consistent with that used in index number literature.

Since the early 1990s, several accounting systems for tracking economy-wide energy efficiency trends have been developed to give energy efficiency indicators as shown in Table 6-1. Different approaches for treatment of time have been adopted by international organizations, national agencies, researchers and analysts. In Canada, OEE has been publishing the annual report titled “Energy Efficiency Trends in Canada” since 1996. In 2008, the 12th edition of the report switched from the non-chaining approach to the chaining approach (OEE, 2008). The Office of Energy Efficiency and Renewable Energy (EERE) of the US Department of Energy (US-DOE) established a national system of indicators to track changes in the energy intensity of US economy and economic sectors over time. Details on the methodology and accounting framework can be found in the EERE website that came on line in 2006 (EERE, 2009) and the chaining approach is adopted. IEA has been developing energy efficiency indicators since 1995 and updates energy efficiency studies compared with 1990 as the base year to track the development. Non-chaining approach is used in IEA.

From the discussion above, we could find that there is no coherent choice between chaining and non-chaining approaches. Given a specific dataset, application of different approaches treating time will lead to different numerical results and therefore different levels of achievement in energy efficiency improvement. Ang and Lee (1994) compare chaining and non-chaining approaches in IDA studies. They point out several disadvantages of the non-chaining approach, such as problems arising from loss of information, and recommended the use of chaining approach when data is available. Till now, this study is still the main methodological paper on chaining and non-chaining approaches in energy-related decomposition analysis. This chapter attempts to extend the study of Ang and Lee (1994). It discusses the advantages and limitations of both the chaining and non-chaining approaches to provide recommendations to practitioners.

Table 6-1. Features of energy efficiency accounting systems/studies

Country/ organisation	Major sectors	Number of sub-sectors	Decomposition approach	Decomposition Method	Treatment of time
Canada	Transport, industrial, residential, commercial, electricity	>100	Additive Decomposition	LMDI-I	Chaining
USA-EERE	Transport, industrial, residential, commercial, electricity	75	Multiplicative Decomposition	LMDI-II	Chaining
New Zealand	Transport, industrial, residential, commercial	25	Additive Decomposition	LMDI-II	Chaining
IEA	Transport, manufacturing, household, service	20	Multiplicative Decomposition	Laspeyres	Non-chaining
Australia	Transport, industrial, residential, services	52	Multiplicative/Additive Decomposition	LMDI-I	Chaining

Sources: Canada, Office of Energy Efficiency (OEE, 2008); USA-EERE, Office of Energy Efficiency and Renewable Energy (EERE, 2009); New Zealand, Energy Efficiency and Conservation Authority (EECA, 2003); International Energy Agency, IEA (2007); and Australia, Sandu and Petchey (2009).

The rest of this chapter is organized as follows. In the next section, we introduce the concepts of chaining and non-chaining approaches. In Section 6.3, we identify the conditions under which the results of chaining and non-chaining approaches will be identical. In Section 6.4, we discuss the advantages and the disadvantages of both chaining and non-chaining approaches, which is followed by Section 6.5 in which the desirable properties of decomposition methods using the chaining approach are studied. In Section 6.6, we present our recommendations and their implications.

6.2 Methodological Review

In this section, we introduce the concepts of chaining and non-chaining approaches. An illustrative example is also given.

6.2.1 Concepts of Chaining and Non-chaining Approaches

Chaining and non-chaining are two different indexing approaches in energy decomposition analysis. If a decomposition analysis is conducted over a time period consisting of a certain number of years using yearly data, say from year 0 to year *T*, decomposition can be conducted based only on the data for the starting year 0 and the ending year *T* without using the data in the intervening years. Alternatively, decomposition can be carried out using the data for every two consecutive years in the time series, i.e. years 0 and 1, 1 and 2, and so on till *T*-1 and *T*. A total of *T* sets of decomposition results can be obtained which can then be "chained" to give the results for the whole time period. The former will be referred to as the non-chaining while the latter the chaining approach.

In some cases, the data set consists of results using the data for years 0 and 1, 0 and 2, and so on till 0 and *T*. In this situation, although time series results are given, we will classify the study as a non-chaining study. In some cases, period-wise results are given. However, the decomposition result of the whole time period is the accumulation of the decomposition results of several continuous time periods which are not yearly results. We will treat these cases as using the chaining approach. When data are available for only two years which are not consecutive, non-chaining analysis is the only choice available to the analyst.

6.2.2 An Illustrative Example

We present a hypothetical case where industry comprises two sectors as shown in Table 6-2 to explain the chaining and the non-chaining approaches. The data in this example are given for three years (Year 0, Year 1 and Year 2). The aggregate energy intensity $I = E/Y$ can be written as

$$I = \sum_{j=1}^{2} \frac{E_j}{Y_j}\frac{Y_j}{Y} = \sum_{j=1}^{2} I_j S_j \tag{6-1}$$

Table 6-2. An illustrative example (arbitrary units)

	Year 0				Year 1				Year 2			
	E^0	Y^0	S^0	I^0	E^1	Y^1	S^1	I^1	E^2	Y^2	S^2	I^2
Sector 1	30	10	0.2	3.0	80	40	0.5	2	120	80	0.67	1.5
Sector 2	20	40	0.8	0.5	16	40	0.5	0.4	12	40	0.33	0.3
Industry	50	50	1.0	1.0	96	80	1.0	1.2	132	120	1	1.1

The change in the aggregate energy intensity is to be decomposed to give estimates of the structure effect and energy intensity effect using the LMDI I method. The decomposition results obtained using both chaining and non-chaining approaches are summarized in Table 6-3. It can be seen that results of chaining and non-chaining approaches are different in both the additive and multiplicative cases.

Table 6-3. Decomposition results obtained using the data in Table 6-2

Treatment of time		Non-chaining	Chaining			Deviation[a]
Time Period		Year [0, 2]	Year [0, 1]	Year [1, 2]	Year $[0, 2]_c$	
Multiplicative Decomposition	D_{tot}	1.10	1.20	0.92	1.10	-
	D_{str}	2.05	1.70	1.22	1.70*1.22=2.07	102.33%
	D_{int}	0.54	0.71	0.75	0.71*0.75=0.53	101.37%
Additive Decomposition	ΔE_{tot}	0.10	0.2	-0.1	0.1	-
	ΔE_{str}	0.75	0.58	0.23	0.58+0.23=0.81	-7.67%
	ΔE_{int}	-0.65	-0.38	-0.33	-0.38-0.33=-0.71	-8.84%

[a]Given by the ratio of the result for chaining minus 1 to that of non-chaining minus 1 in the case of multiplication decomposition, and by the result for non-chaining minus that of chaining and then divided by the result of non-chaining in the ease of additive decomposition.

6.3 Transitivity Test

Transitivity test has been studied in Section 3.4. In INP, there are only a few specific index numbers that satisfy the transitivity test. Funke et al. (1979) conclude that a function satisfies monotonicity axiom, linear homogeneity axiom, identity axiom, commensurability axiom and the transitivity test if and only if it is the Cobb-Douglas price index. If we take the energy intensity effect in Section 6.2.2 as an example, the formula of the Cobb-Douglas index is as follows

$$\begin{aligned} D_{\text{int}}^{0,T} &= \prod_{j=1}^{n}\left(\frac{I_j^T}{I_j^0}\right)^{w_j} = \prod_{j=1}^{n}\left(\frac{I_j^1}{I_j^0}\right)^{w_j}\cdot\left(\frac{I_j^2}{I_j^1}\right)^{w_j}\cdots\left(\frac{I_j^T}{I_j^{T-1}}\right)^{w_j} \\ &= \prod_{t=0}^{T-1}\left(\prod_{j=1}^{n}\left(\frac{I_j^{t+1}}{I_j^t}\right)^{w_j}\right) = \prod_{t=0}^{T-1} D_{\text{int}}^{t,t+1} \end{aligned} \tag{6-2}$$

where w_j is the weight in the Cobb-Doulas index with $\sum_j w_j = 1$.

The Divisia index decomposition methods are supposed to satisfy the transitivity test before approximation. This property is shown in Eq. (6-3) and (6-4):

$$\Delta V_i^{0,T} = \sum_j \int_0^T V_j d\ln x_{j,i} = \sum_j \sum_{t=0}^{T-1} \int_t^{t+1} V_j d\ln x_{j,i} = \sum_{t=0}^{T-1} \Delta V_i^{t,t+1} \tag{6-3}$$

$$D_{x_i}^{0,T} = \exp\left\{ \int_0^T \sum_{j=1}^m \frac{V_j}{V} \frac{d\ln(x_{j,i})}{dt} \right\} = \exp\left\{ \sum_{t=0}^{T-1} \int_t^{t+1} \sum_{j=1}^m \frac{V_j}{V} \frac{d\ln(x_{j,i})}{dt} \right\} = \prod_{t=0}^{T-1} D_{x_i}^{t,t+1} \tag{6-4}$$

However in empirical studies, only discrete data are given for the time intervals, hence only approximated forms of Eq. (6-3) and Eq. (6-4) are given based on the weight function. The various IDA methods linked to the Divisia index are differentiated primarily by the weight functions in Eq. (6-3) and Eq. (6-4). Different IDA methods assume different hypothesized paths for the time series data. The hypothesized paths for additive LMDI I, S/S, Laspeyres and Paasche methods are elaborated below.

The hypothesized paths for the additive LMDI I are as follows:

$$\ln x_{j,i}^t - \ln x_{j,i}^0 = t(\ln x_{j,i}^1 - \ln x_{j,i}^0) \tag{6-5}$$

To make the expression simple, we use 0 and 1 to represent the base year 0 and the target year T respectively. Since $V_j = x_{j,1} \cdot x_{j,2} \cdot \ldots \cdot x_{j,n}$, then

$$\sum_i \ln x_{j,i}^t - \sum_i \ln x_{j,i}^0 = t(\sum_i \ln x_{j,i}^1 - \sum_i \ln x_{j,i}^0)$$
$$\Rightarrow \ln V_j^t - \ln V_j^0 = t(\ln V_j^1 - \ln V_j^0) \tag{6-6}$$
$$\Rightarrow V_j^t = V_j^0 \cdot (\frac{V_j^1}{V_j^0})^t$$

The effect associated with factor *i* is given by

$$\Delta V_{x_i}^{0,1} = \sum_j \int_0^1 V_j d\ln x_{j,i} = \sum_j \int_0^1 V_j^0 \cdot (\frac{V_j^1}{V_j^0})^t \cdot (\ln x_{j,i}^1 - \ln x_{j,i}^0) dt$$
$$= \sum_j L(V_j^1, V_j^0) \cdot \ln \frac{x_{j,i}^1}{x_{j,i}^0} \tag{6-7}$$

which is the LMDI I method.

The hypothesized paths for S/S are as follows:

$$x_{j,i}^t - x_{j,i}^0 = t(x_{j,i}^1 - x_{j,i}^0) \tag{6-8}$$

Since $V_j = x_{j,1} \cdot x_{j,2} \cdot \ldots \cdot x_{j,n}$, then

$$V_j^t = \prod_i [x_{j,i}^0 + t(x_{j,i}^1 - x_{j,i}^0)] \tag{6-9}$$

We take factor x_1 as an example. The effect associated with factor x_1 is

$$\Delta V_{x_1}^{0,1} = \sum_j \int_0^1 V_j d\ln x_{j,1} = \sum_j \int_0^1 \prod_{i=2}^n [x_{j,i}^0 + t(x_{j,i}^1 - x_{j,i}^0)] \cdot (x_{j,1}^1 - x_{j,1}^0) dt$$
$$= \sum_j (\frac{V_j^0}{x_{j,1}^0} \Delta x_{j,1} + \frac{1}{2} \sum_{p \neq 1}^n \frac{V_j^0}{x_{j,1}^0 \cdot x_{j,p}^0} \Delta x_{j,1} \Delta x_{j,p} \tag{6-10}$$
$$+ \frac{1}{3} \sum_{p \neq q \neq i}^n \frac{V_j^0}{x_{j,1}^0 \cdot x_{j,p}^0 \cdot x_{j,q}^0} \Delta x_{j,1} \Delta x_{j,p} \Delta x_{j,q} + \ldots + \frac{\Delta x_{j,1} \Delta x_{j,2} \ldots \Delta x_{j,n}}{n})$$

which is the S/S method.

The hypothesized paths for additive and multiplicative Laspeyres are as follows:

$$x_{j,i}^t = \begin{cases} x_{j,i}^0 & t \in [0,1) \\ x_{j,i}^1 & t = 1 \end{cases} \tag{6-11}$$

$$x_{j,p}^t = x_{j,p}^0 \text{, where } p \neq i \tag{6-12}$$

Since $V_j = x_{j,1} \cdot x_{j,2} \cdot \ldots \cdot x_{j,n}$, then

$$V_j^t = \begin{cases} \prod_{p=1}^{n} x_{j,p}^0 & t \in [0,1) \\ \prod_{p \neq i}^{n} x_{j,i}^1 \cdot x_{j,p}^0 & t = 1 \end{cases} \tag{6-13}$$

The effect associated with factor *i* is given by

$$\begin{aligned} \Delta V_{x_i}^{0,1} &= \sum_j \int_0^1 V_j d \ln x_{j,i} = \sum_j \int_0^1 \prod_{p=1}^{n} x_{j,p}^t \cdot \frac{dx_{j,i}^t}{x_{j,i}^t} \\ &= \sum_j dV_j^t = \sum_j \prod_{p \neq i}^{n} x_{j,p}^0 \cdot \Delta x_{j,i} \end{aligned} \tag{6-14}$$

$$\begin{aligned} D_{x_i}^{0,1} &= \exp\left\{ \int_0^1 \sum_{j=1}^{m} \frac{V_j}{V} \frac{d \ln x_{j,i}}{dt} \right\} = \exp\left\{ \int_0^1 \sum_{j=1}^{m} \frac{V_j}{V} \frac{dx_{j,i}}{x_{j,i}} \right\} \\ &= \exp\left\{ \int_0^1 \frac{dV}{V} \right\} = \frac{\sum_j \prod_{p \neq i}^{n} x_{j,i}^1 \cdot x_{j,p}^0}{\sum_j \prod_{p=1}^{n} x_{j,p}^0} \end{aligned} \tag{6-15}$$

Eq. (6-14) and Eq. (6-15) are the additive and multiplicative Laspeyres methods respectively.

The hypothesized paths for additive and multiplicative Paasche are as follows:

$$x_{j,i}^{t} = \begin{cases} x_{j,i}^{0} & t \in [0,1) \\ x_{j,i}^{1} & t = 1 \end{cases} \tag{6-16}$$

$$x_{j,p}^{t} = x_{j,p}^{1}, \text{ where } p \neq i \tag{6-17}$$

Since $V_j = x_{j,1} \cdot x_{j,2} \cdot \ldots \cdot x_{j,n}$, then

$$V_j^t = \begin{cases} \prod_{p \neq i}^{n} x_{j,i}^{0} \cdot x_{j,p}^{1} & t \in [0,1) \\ \prod_{p=1}^{n} x_{j,p}^{1} & t = 1 \end{cases} \tag{6-18}$$

The effect associated with factor *i* is given by

$$\begin{aligned} \Delta V_{x_i}^{0,1} &= \sum_j \int_0^1 V_j d\ln x_{j,i} = \sum_j \int_0^1 \prod_{p=1}^{n} x_{j,p}^{t} \cdot \frac{dx_{j,i}^{t}}{x_{j,i}^{t}} \\ &= \sum_j dV_j^t = \sum_j \prod_{p \neq i}^{n} x_{j,p}^{1} \cdot \Delta x_{j,i} \end{aligned} \tag{6-19}$$

$$\begin{aligned} D_{x_i}^{0,1} &= \exp\left\{ \int_0^1 \sum_{j=1}^{m} \frac{V_j}{V} \frac{d\ln x_{j,i}}{dt} \right\} = \exp\left\{ \int_0^1 \sum_{j=1}^{m} \frac{V_j}{V} \frac{dx_{j,i}}{x_{j,i}} \right\} \\ &= \exp\left\{ \int_0^1 \frac{dV}{V} \right\} = \frac{\sum_j \prod_{p=1}^{n} x_{j,p}^{1}}{\sum_j \prod_{p \neq i}^{n} x_{j,i}^{1} \cdot x_{j,p}^{0}} \end{aligned} \tag{6-20}$$

Eq. (6-19) and Eq. (6-20) are the additive and multiplicative Paasche methods respectively.

When the actual data satisfy the hypothesized path for a specific IDA method, it passes the transitivity test and the chaining and non-chaining approaches will give the same results. We use a case to this condition and situation. Assume that industry comprises two sectors as shown in Table 6-4 and the data are available for three years (Year 0, Year 1 and Year 2). The energy consumption can be written as:

$$E = \sum_{j=1}^{2} Y_j \frac{E_j}{Y_j} = \sum_{j=1}^{2} Y_j I_j \tag{6-21}$$

The three year energy intensity data and activity data satisfy the hypothesized path under additive LMDI I for Eq. (6-21). This means

$$\ln Y_j^1 - \ln Y_j^0 = 0.5 \cdot (\ln Y_j^2 - \ln Y_j^0) \tag{6-22}$$

$$\ln I_j^1 - \ln I_j^0 = 0.5 \cdot (\ln I_j^2 - \ln I_j^0) \tag{6-23}$$

The decomposition results obtained using both chaining and non-chaining approaches are summarized in Table 6-5. The second column gives the non-chaining decomposition results, the third column shows the decomposition results between year 0 and year 1, the fourth column shows the decomposition results between year 1 and year 2, and the last column gives the chaining decomposition results. We find that the decomposition results of chaining and non-chaining approaches are the same and the transitivity test is satisfied.

Table 6-4. An illustrative example for IDA (arbitrary units)

Year	0			T_1			T_2		
Sector	E_j	Y_j	I_j	E_j	Y_j	I_j	E_j	Y_j	I_j
Sector 1	150	10	15	273.86	14.14	19.36	500	20	25
Sector 2	1200	40	30	1449.14	44.72	32.40	1750	50	35
Industry	1350	50	27	1723	58.86	29.27	2250	70	32.14

Table 6-5. Results of decomposition using the data in Table 6-4 (Additive LMDI I)

	$[0,2]_{NC}$	[0,1]	[1,2]	$[0,2]_{C}$
ΔE_{act}	526.79	218.66	308.13	218.66+308.13= 526.79
ΔE_{int}	373.21	154.34	218.87	154.34+218.87= 373.21
ΔE_{tot}	900.00	373.00	527.00	373+527=900

In summary, the transitivity test is a very restrictive test. Under normal circumstances with a given set of real data, no IDA methods will pass the transitivity test. As a result, the application of chaining and non-chaining approaches will lead to different decomposition results. There is a need to have a better understanding among practitioners of the underlying issues and the implications of the choice they make between chaining and non-chaining analysis.

6.4 Comparison between Chaining and Non-chaining Approaches

In this section, we discuss the advantages and the disadvantages of both chaining and non-chaining approaches and address some of the issues under three aspects: representativeness, result reliability, and flexibility.

6.4.1 Representativeness

Since different IDA methods assume different hypothesized paths of the time series data, when the hypothesized path for an IDA method is close to the real data change, the IDA method would be preferred. In application studies, it is difficult to select a specific IDA method based on the real data change. However, using the chaining approach, the hypothesized path is expected to be closer to the real data change compared to the non-chaining approach. The reason is that more information is available for the chaining approach using time series data compared to the non-chaining approach which uses only base year and target year data.

We use the energy intensity data for United States “Wood Product Manufacturing” sub-sector (1994-2004) in the manufacturing sector as an example to illustrate the path approximation advantage of chaining approach. The data are reported in the U.S. Department of EERE and are shown in Table B-1 in Appendix B. The aggregate energy intensity change in the manufacturing sector is decomposed into two factors: energy intensity effect and structure effect. The energy intensity (I) is the ratio of delivered energy consumption to value-added, and the unit of measure is Btu/$. Using the additive LMDI I method, the energy intensity path of this subsector follows Eq. (6-5). To make the expression simple, we use 0 and 1 to represent the starting point (0) and the ending point (T) respectively. In this case, 0 stands for year 1994 and 1 stands for year 2004. In Figure 6-1, there are 11 real data points, which are used to show the path of energy intensity from year 1994 to year 2004.

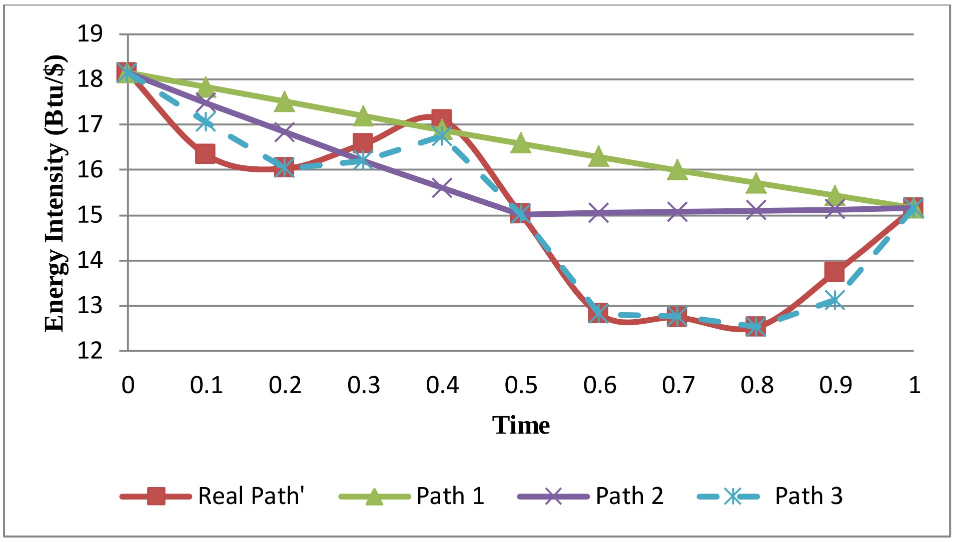

Figure 6-1. Line integral curve of energy intensity for calculating the energy intensity effect using LMDI I for US “Wood Product Manufacturing” sub-sector, 1994-2004.

In Figure 6-1, we treat the smoothed path of the data points as a “real path” of energy intensity changing from 1994 to 2004. When we only use the data point in 1994 and 2004 as in the non-chaining approach to do the decomposition analysis, the line integral path is given by path 1, which is very different from the ‘real path’. When adding one data point in year 1999 and the time intervals become smaller, the line integral path is given by path 2. Using 2 more data points (year 1996 and year 2002), each decomposition time interval becomes even smaller, and the line integral path is given by path 3, which is the closest to the “real path” among the three paths.

The analysis above shows that, using the chaining approach, the real path is closer to the hypothesized path when more data points are used. This means that the decomposition results using the chaining approach are more representative compared to the non-chaining approach. This advantage is more significant when the original time interval from year 0 to year *T* is large.

Despite the above advantage, it is also necessary to highlight a limitation of the chaining approach. When the data oscillate, the chaining approach may have problems in result interpretation. In index number theory, Szulc (1983) points out that when prices or quantities oscillate ("bounce"), chaining can lead to considerable index drift. That is, if after several periods of bouncing, prices and quantities return to their original levels, a chained index will not normally return to zero or one (zero for additive measure, one for multiplicative measure). The bounce problem also exists in IDA when chaining decomposition is used.

We use the US manufacturing sector example (the data set is shown in Table B-2 and Table B-3 in Appendix B) to explain this problem and replace the manufacturing sector data in 1993 with the data in 1990 so that the data of these two years are exactly the same. It is reasonable to assume that the effect of each factor should be equal to 0 for additive and 1 for multiplicative from 1990 to 1993. Table 6-6 and Table 6-7 show the decomposition results of the energy intensity effect obtained using LMDI I, LMDI II, AMDI, Fisher (S/S) and Laspeyres for additive and multiplicative decomposition respectively. From Table 6-6, we find that the estimates of the effect of energy intensity are 0 in 1993 in the non-chaining approach, whereas they are not equal to 0 in the chaining approach. The same conclusion can be reached based on the results for multiplicative decomposition in Table 6-7.

Table 6-6. Decomposition results of US manufacturing sector using five decomposition methods: additive energy intensity effect, 1990-1995

Treatment of time	Chaining					Non-chaining				
ID methods	LMDI II	LMDI I	AMDI	S/S	Las	LMDI II	LMDI I	AMDI	S/S	Las
1990	0	0	0	0	0	0	0	0	0	0
1991	-0.303	-0.303	-0.303	-0.303	-0.289	-0.303	-0.303	-0.303	-0.303	-0.290
1992	-0.939	-0.939	-0.939	-0.940	-0.900	-0.930	-0.930	-0.929	-0.933	-0.860
1993	-0.009	-0.009	-0.010	-0.007	-0.107	0	0	0	0	0
1994	-1.795	-1.794	-1.795	-1.811	-1.450	-1.785	-1.785	-1.785	-1.804	-1.557
1995	-1.275	-1.275	-1.275	-1.289	-0.841	-1.177	-1.177	-1.179	-1.191	-1.086

The influence of the “bounce problem” can be reduced when an IDA method passes the time-reversal test. According to the definition of the, when the data path is symmetrical, the ‘bounce problem’ does not exist when an IDA method passing the test is used. Since LMDI I, LMDI II, AMDI and S/S (Fisher) satisfy the time reversal time, it is expected that the decomposition results of the four methods are closer to 0 (or 1) compared to the Laspeyres method, which does not satisfy the time-reversal time. For example, in Table 6-7, in 1993, the energy intensity effect is 0.9997 for LMDI I, 0.9996 for the Fisher method, but 1.0049 for the Laspeyres method. This shows possessing the time-reversal property helps to reduce the influence of the “bounce problem”.

Although the “bounce problem” is inherent in the chaining approach, it does not necessarily mean that the non-chaining approach is superior to the chaining approach. We again use the energy intensity data for United States “Wood Product Manufacturing” sub-sector (1994-2004) in the manufacturing sector as an example. We replace the energy intensity data in 2004 by those of

1994 so that the data of those two years are exactly the same. The hypothesized paths of the chaining and non-chaining approaches to calculating the energy intensity effect are different and they are illustrated in Figure 6-2.

Table 6-7. Decomposition results of US manufacturing sector using five decomposition methods: multiplicative energy intensity effect, 1990-1995

Treatment of time	Chaining					Non-haining				
IDA methods	LMDI II	LMDI I	AMDI	Fisher	Las	LMDI II	LMDI I	AMDI	Fisher	Las
1990	1	1	1	1	1	1	1	1	1	1
1991	0.9785	0.9785	0.9785	0.9785	0.9792	0.9785	0.9785	0.9785	0.9785	0.9792
1992	0.9347	0.9347	0.9348	0.9347	0.9366	0.9350	0.9351	0.9351	0.9351	0.9382
1993	0.9997	0.9997	0.996	0.9996	1.0049	1	1	1	1	1
1994	0.8773	0.8774	0.8774	0.8775	0.8924	0.8776	0.8777	0.8777	0.8778	0.8880
1995	0.9125	0.9125	0.9126	0.9126	0.9328	0.9161	0.9161	0.9162	0.9155	0.9219

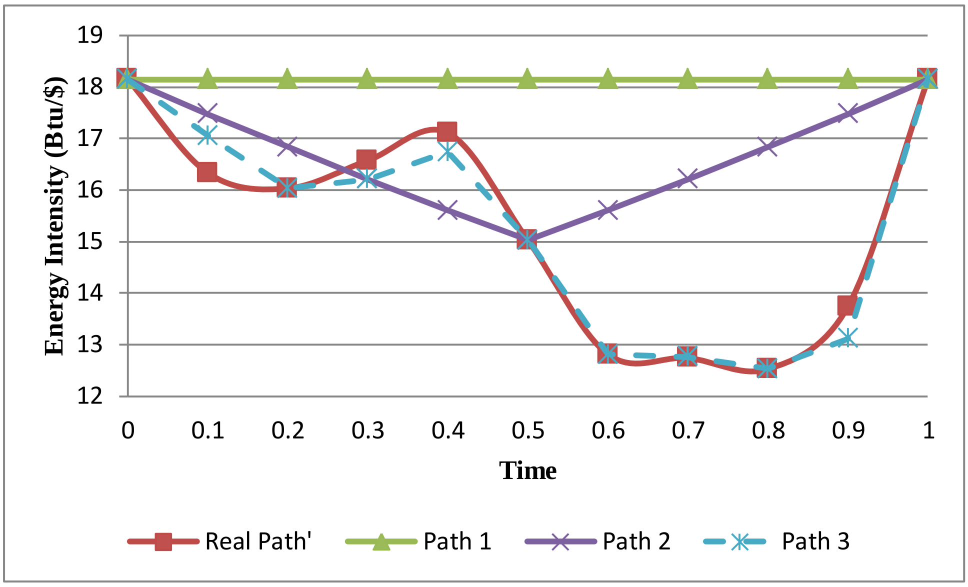

Figure 6-2. Line integral curve of energy intensity for calculating the energy intensity effect using LMDI I for US “Wood Product Manufacturing” sub-sector, 1994-2004 (bounce problem)

When we only use the data of 1994 and 2004 as in the non-chaining approach in the decomposition analysis, the line integral path is given by path 1, which is quite different from the “real path”. Adding one data point (year 1999) and the time intervals becoming shorter, the line integral path is given by path 2. Adding two more data points (year 1996 and year 2002), and the line integral path is given by path 3, which is the best among these three paths. As discussed above, the chaining approach is based on a path which is closer to the “real path” and it should be preferred due to its representativeness. In the specific “bounce problem”, the approximation of the non-chaining approach is easy to interpret but this does not necessarily mean that it is generally preferred.

6.4.2 Result Reliability

Ang and Lee (1994) point out that in general, the decomposition results given by the chaining approach are less dependent on the IDA method used and the yearly sum estimate of the residual term is also much smaller as compared to the non-chaining approach. The findings of other studies, such as that on primary energy in US manufacturing (1960-1995) as reported in Greening et al. (1997), also support this argument.

We attempt to explain why the decomposition results given by the chaining approach are less IDA method dependent. In Section 6.3, we show that different IDA methods are based on different hypothesized paths of the time series data. When time series data are used, we can break the entire time span into shorter time intervals in the chaining approach. In the individual time intervals, the hypothesized paths of different IDA methods become closer to each other. On the contrary, the entire time span between the base year and target year is the time interval used in the non-chaining approach. Therefore, the decomposition results given by different IDA studies are closer using the chaining approach.

To illustrate, we again use the data for the United States manufacturing sector from 1990 to 2004 given in Appendix B. The manufacturing sector is disaggregated into 18 subsectors. We use additive and multiplicative LMDI I, LMDI II, AMDI, S/S (S/S for additive and Fisher for multiplicative) and Laspeyres decomposition methods. The aggregate energy intensity changes are decomposed into two factors, structure effect and energy intensity effect, similar to the example in Section 6.2.2. The additive decomposition results are shown in Figure 6-3- Figure 6-6. The decomposition results for multiplicative measure are shown in Appendix C.

From Figure 6-3 to Figure 6-6, we can see that the decomposition results given by the five IDA methods are closer to each other using the chaining approach than the non-chaining approach.

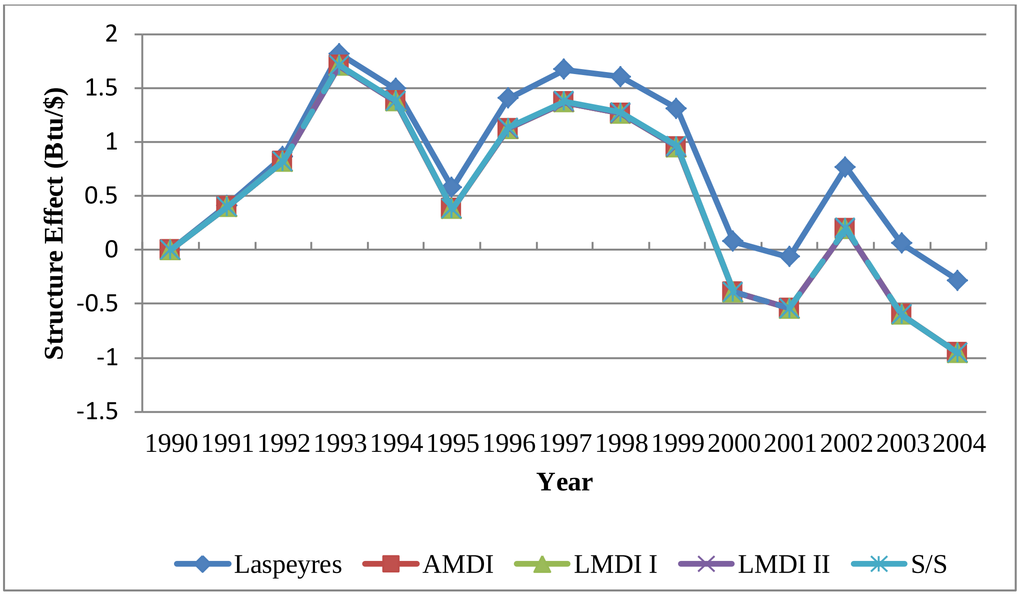

Figure 6-3. Decomposition results for US manufacturing sector, 1990-2004: structure effect, chaining (additive decomposition).

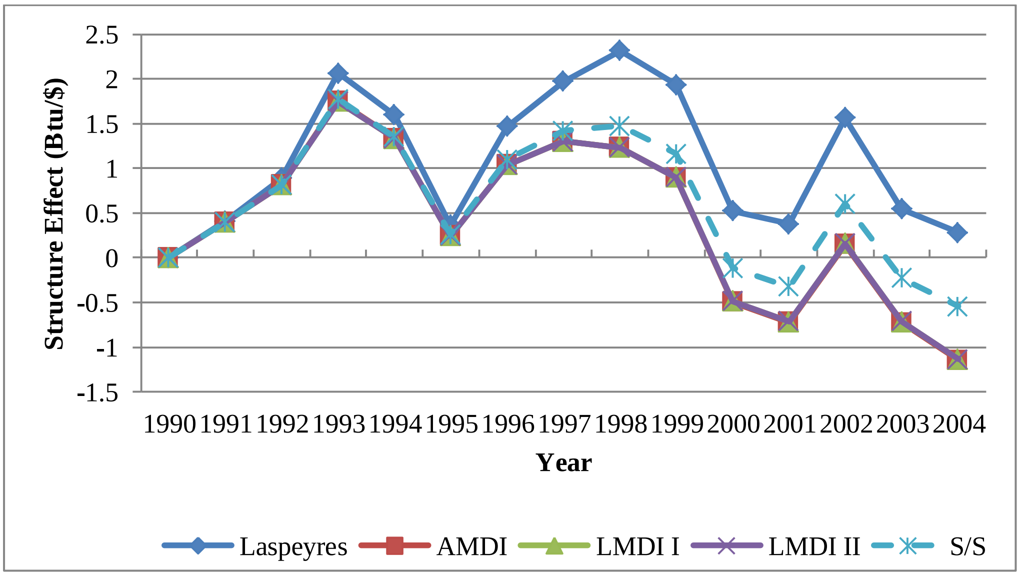

Figure 6-4. Decomposition results for US manufacturing sector, 1990-2004: structure effect, non-chaining (additive decomposition).

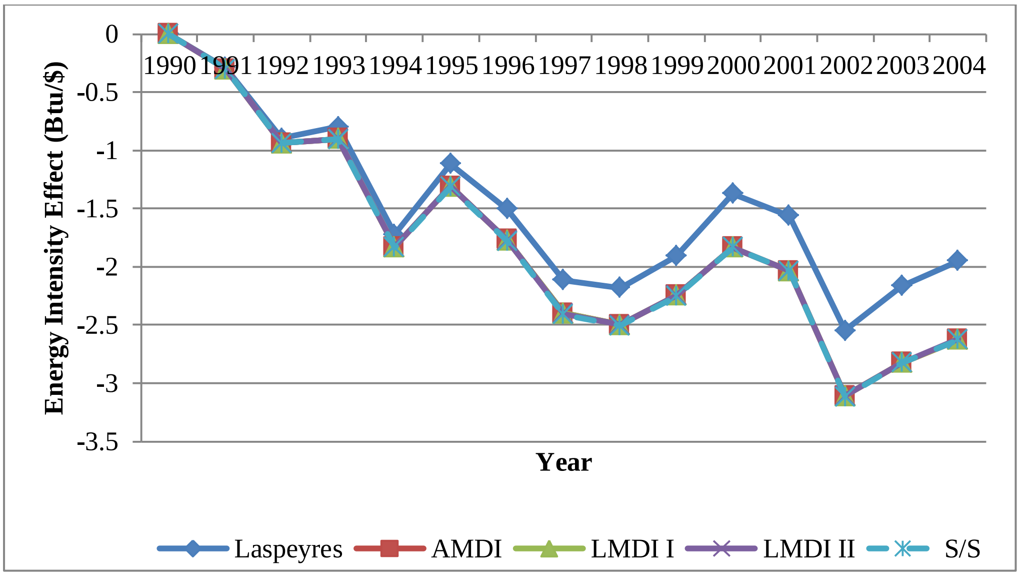

Figure 6-5. Decomposition results for US manufacturing sector, 1990-2004: energy intensity effect, chaining (additive decomposition).

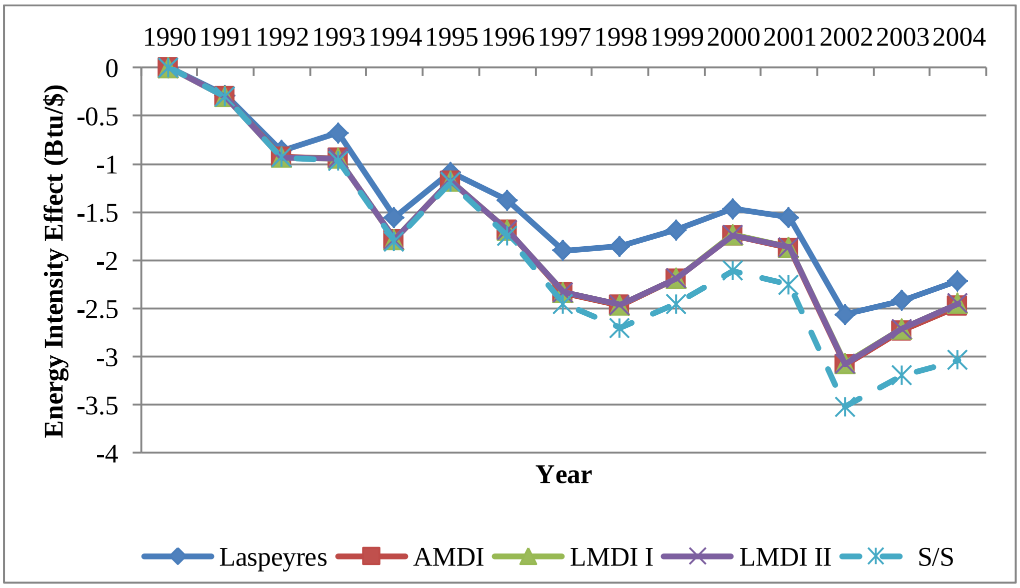

Figure 6-6. Decomposition results for US manufacturing sector, 1990-2004: energy intensity effect, non-chaining (additive decomposition).

6.4.3 Flexibility

It is clear from the discussions presented above that the non-chaining approach is associated with a pre-specified base year. Hence the non-chaining approach is also referred to as the "fixed base" indexing approach. The choice of base year in a specific study could also be rather arbitrary and an update is normally needed after a few years. In the chaining approach, no base year needs to be specified and studies can be conducted for any time period, i.e. for any portion of time-series. Therefore, the chaining approach is more flexible than the non-chaining approach in this aspect.

For the non-chaining approach, a change of the base year will lead to a re-calculation of all the decomposition results, while for the chaining approach, a change of the base year will lead to re-cumulation of the relative change between consecutive years t and t+1 but will not affect the relative change itself. For example, assuming data are available for n years from year 1 exclusive of the data for base year 0. Using either chaining or non-chaining approaches, n decomposition analyses are necessary to get the decomposition results of n years. If the base year changes after the initial calculations, the non-chaining approach needs to re-calculate all the decomposition results; while in the chaining approach, n consecutive years decomposition analyses will provide the decomposition results of $(1+n)*n/2$ times span (any portion of the time-series).

6.5 Check for Desirable Properties

Ang et al. (2002) use four tests in index number theory to determine the desirability of a decomposition method: factor-reversal, time-reversal, proportionality and aggregation tests, and the focus is on non-chaining analysis. In this section, we study whether the desirable properties still exist when the chaining approach is used.

6.5.1 Factor-reversal Test

The factor reversal test suggests that the product of estimations of the predefined factors in an aggregate should give the same value index as the aggregate change. In other words, the factor reversal test deals with the residual term issue. The following equations can be deducted to show that the chaining approach does not affect the factor-reversal property.

$$\Delta V_{rsd}^{t,t+1} = 0 \Rightarrow \Delta V_{rsd}^{0,T} = \sum_{t=0}^{T-1} \Delta V_{rsd}^{t,t+1} = 0 \tag{6-24}$$

$$D_{rsd}^{t,t+1} = 1 \Rightarrow D_{rsd}^{0,T} = \prod_{t=0}^{T-1} D_{rsd}^{t,t+1} = 1 \tag{6-25}$$

6.5.2 Time-reversal Test

The time reversal test expresses the ability of an index calculated from past to present as exactly reciprocal to the one calculated from present to past. Passing this test means the results are symmetrical. Take factor x_i as an example, the following equations can be derived to show that the chaining approach does not affect the time-reversal property.

$$\Delta V_{x_i}^{t,t+1} = -\Delta V_{x_i}^{t+1,t} \Rightarrow$$

$$\begin{aligned}\Delta V_{x_i}^{0,T} &= \sum\nolimits_{t=0}^{T-1} \Delta V_{x_i}^{t,t+1} = -\sum\nolimits_{t=0}^{T-1} \Delta V_{x_i}^{t+1,t} \\ &= -\sum\nolimits_{t=T-1}^{0} \Delta V_{x_i}^{t+1,t} = -\Delta V_{x_i}^{T,0}\end{aligned} \tag{6-26}$$

$$\begin{aligned}&D_{x_i}^{t,t+1} = \frac{1}{D_{x_i}^{t+1,t}} \Rightarrow \\ &D_{x_i}^{0,T} = \prod_{t=0}^{T-1} D_{x_i}^{t,t+1} = \prod_{t=0}^{T-1} \frac{1}{D_{x_i}^{t+1,t}} = \frac{1}{\prod_{t=0}^{T-1} D_{x_i}^{t+1,t}} = \frac{1}{\prod_{t=T-1}^{0} D_{x_i}^{t+1,t}} = \frac{1}{D_{x_i}^{T,0}}\end{aligned} \tag{6-27}$$

6.5.3 Proportionality Test

Passing the proportionality test means that if the current value of a factor is multiplied by a positive constant *k*, then the new index should be equal to the old index multiplied by *k*. Taking x_i factor as an example and we assume that in each time interval factor x_i is multiplied by a positive constant k^{t+1}. Then at the target year, the value of factor x_i is multiplied by a positive constant $\prod_{t=0}^{T\text{-}1} k^{t+1}$. We can see that the chaining approach does not affect the factor-reversal property:

$$\begin{aligned}&D_{x_i}^{t,t+1}(k^{t+1} \cdot x_i^{t+1}) = k^{t+1} \cdot D_{x_i}^{t,t+1}(x_i^{t+1}) \\ &\Rightarrow D_{x_i}^{0,T}(\prod_{t=0}^{T\text{-}1} k^{t+1} \cdot x_i^{T}) = \prod_{t=0}^{T\text{-}1} D_{x_i}^{t,t+1}(k^{t+1} \cdot x_i^{t+1}) \\ &\Rightarrow \prod_{t=0}^{T\text{-}1} (k^{t+1} \cdot D_{x_i}^{t,t+1}(x_i^{t+1})) = \prod_{t=0}^{T\text{-}1} k^{t+1} \cdot D_{x_i}^{0,T}(x_i^{T})\end{aligned} \tag{6-28}$$

6.5.4 Consistency in Aggregation Test

An index formula is consistent in aggregation if the value of the index calculated in two steps necessarily coincides with the value of the index as calculated in an ordinary way in a single step (Ang and Liu (2001)). The chaining approach has a problem of determining at which step to cumulate the decomposition results over time. If we cumulate the results at the second step, it is obvious that the chaining approach will not influence the consistency aggregation property of the decomposition methods. Using the chaining method, for the additive approach, the property of consistency in aggregation remains the same, no matter whether we cumulate the results at the first step or at the second step. The proof is provided in Appendix D. However, for the multiplicative decomposition, if we cumulate the results at the first step, the property of aggregation consistency is not satisfied. Therefore, we recommend cumulating the results at the second step when using the chaining approach.

6.6 Conclusion

In this chapter, we extended the discussion in Ang and Lee (1994) on chaining and non-chaining decomposition and provided a more complete picture about these two approaches to decomposition analysis. We began by reviewing the development of chaining and non-chaining approaches. We then identified the conditions under which the results of chaining and non-chaining approaches will be identical.

We also discussed chaining and non-chaining approaches in terms of “representativeness”, “results reliability”, and “flexibility”. Based on our research, we believe that chaining analysis should be preferred as the corresponding results provide a more realistic measure of the real changes, such as in energy efficiency, over time. This is especially true when the study is over a long time period. In addition, the decomposition results given by the chaining approach are less dependent on the IDA method used. Moreover, chaining makes full use of the data available, and it is more flexible in terms of application.

The selection of either chaining or non-chaining is also determined by the data available and the data characteristics. From the analysis above, we concluded that the chaining approach is preferred when the data oscillation problem is not frequent or significant. When the data bounce problem is significant and the time span is not large, the non-chaining approach may be preferred.

CHAPTER 7: Conclusion

In this thesis, we studied several methodological issues of IDA, from theory underpinning to methodology recommendations. A literature review was presented to illustrate the development of IDA both in terms of methodology and application. Linkages and differences between INP and IDA were studied based on both multiplicative and additive decomposition approaches. Relationships between the Laspeyres-based IDA methods and the Shapley value in game theory were formalized through defining the characteristic functions in the Shapley value. Properties and linkages of Divisia-based IDA methods were studied with the LMDI I method recommended. Advantages and disadvantages of chaining and non-chaining approaches were examined and recommendations were provided.

The main findings and implications of each chapter are summarized and future research topics are suggested.

7.1 Main Findings and Contributions

This thesis mainly dealt with the methodological issues of IDA. The research study reported in this thesis has contributed to IDA and its application in five respects.

First, we conducted a comprehensive literature survey for IDA and brought the 2000 survey up to date. In Chapter 2, we studied the main features of the IDA studies on both methodology and application: application area, indicator type, decomposition approach, decomposition methods, treatment of time and level of disaggregation. Research gaps were identified from the review and evaluations of the developments of IDA through time have been made based on this comprehensive review.

Second, we summarized the similarities and the differences between IDA and INP. IDA and INP have a close relationship in terms of both methods and properties. Some main IDA methods and tests are derived from INP. From the study of the similarities between IDA and INP, we provided the theoretical foundations to support the derivations of IDA methods and tests from INP. In addition, we summarize the existing tests to evaluate IDA methods, identify the problems of tests in IDA studies and introduced three new tests to better reflect whether a method is effective in performing decomposition analysis. The summary of criteria will enable researchers to better understand and apply IDA methods corresponding to different situations and data sources.

Third, we formalized the relationships between the Laspeyres-based methods and the Shapley value in game theory. It was shown that linkage can be established through defining the characteristic function in the Shapley value, and that the "jointly created and equally distributed" principle proposed in Sun (1998) was equivalent to the fair allocation principle in the Shapley value. Following this line of reasoning, the principle of the Shapley value can be further extended to cover some other IDA methods in a unified and coherent manner. Furthermore, S/S and the generalized Fisher methods were recommended as the preferred additive Laspeyres-based IDA method and multiplicative Laspeyres-based IDA method respectively.

Fourth, we studied the properties and linkages of Divisia-based IDA methods. We found that most additive Divisia-based IDA methods, including AMDI and LMDI II, collapse to LMDI I after applying the "proportionally distributed by sub-category" principle to the residual terms, which is methodologically interesting. The principle provided a formal linkage between AMDI and LMDI I which had previously been thought to be unrelated. Furthermore, with this linkage, the problem that AMDI fails when there are zero values in the data can now be resolved. In multiplicative decomposition, we proved that the multiplicative LMDI I method was the only perfect method that satisfied the consistency in aggregation test. In addition, we recommended that the LMDI I method was the preferred Divisia-based IDA method.

Finally, we discussed the advantages and disadvantages of chaining and non-chaining approaches and provided recommendations. We reviewed the development of chaining and non-chaining approaches in IDA studies. We identified the conditions under which the results of chaining and non-chaining approaches would be identical. We then discussed and compared chaining and non-chaining approaches on "representativeness", "results reliability", and "flexibility" aspects. We found that the decomposition results given by the chaining approach are less dependent on method selection. Moreover, the chaining approach made full use of the data available. In addition, the chaining approach was more flexible in terms of application. We recommended that the chaining approach should be preferred as the corresponding results provide a more realistic measure of the factor effects over time. This is especially true when the study was over a long time period.

7.2 Areas of Future Research

Despite the contributions described above, there are several areas we think future research can be conducted. These future research areas are summarized below.

In Chapter 5, we recommended distribution of the residual terms in the principle of "proportionally distribute by sub-category" and established linkages among various Divisia-based IDA methods in additive decomposition. Since it is difficult to measure the distribution of residual terms at sub-category level in multiplicative, similar properties and linkages have not been established in the multiplicative case. Properties and linkages among Divisia-based IDA methods in multiplicative decomposition can be a topic for future research.

In Chapter 3, we studied the theoretical foundation of IDA from the viewpoint of INP in economics. In Chapter 4, we formalized the linkages between Laspeyres-based IDA methods and the Shapley value in game theory. There are similarities and differences among INP, IDA and cooperative game theory. A comprehensive and systematic inter-disciplinary research study about these three areas will help to consolidate IDA studies from some new perspectives.

In IDA, it is generally agreed that using physical activity measures provides a better estimate of the real changes in energy efficiency, compared with that based on economic activity measures. When structure effect exists, there is a requirement that different sub-sectors should have the same unit to calculate the structural effect. Therefore, physical indicators are seldom used in IDA and economic activity measures are used as the activity indicators in general. With the limitation of consistent units, how to deal with physical indicators to provide a better estimate of the real energy efficiency change has been a topic in IDA.

Economy-wide energy efficiency tracking and monitoring is an important topic in energy and environment studies. There are many differences among the existing accounting systems for tracking economy-wide energy efficiency trends. In this thesis, we recommended using chaining LMDI I method in the system design. Further studies are needed to provide a greater uniformity in the design and application of such systems.

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Appendix A: Proof of the Identicalness between Laspeyres-based Shapley Value and the S/S Method

We establish the proof in three cases. First we show that the Shapley decomposition for the Laspeyres index form is the same as the decomposition from the S/S method. Next we show that application of the Shapley decomposition with characteristic functions given in the Laspeyres and Paasche index forms gives the same Shapley decomposition results. Lastly we show that the Shapley decomposition with the characteristic function given in the general form in produces only a unique set decomposition result irrespective of the value of α. This is also the set decomposition result given by the S/S method.

Shapley decomposition for the Laspeyres index form is identical to the S/S method

Applying the characteristic function in the Laspeyres index form given in Table 4-1, the corresponding Shapley value for the i^{th} factor is

$$\phi_i(v) = \sum_{j=1}^{m}\sum_{r=1}^{n}\sum_{\substack{R\subseteq N\\ i\in R\\ |R|=r}}\sum_{K\subseteq R}(-1)^{r-|K|}v_j(K)/r = \sum_{j=1}^{m}\sum_{r=1}^{n}\sum_{\substack{R=\{l_1,l_2,\cdots,l_r\}\\ i=l_1}}\sum_{K\subseteq R}(-1)^{r-|K|}v_j(K)/r.$$

Also from Sun (1998), the S/S value of the i^{th} factor is

$$\sum_{j=1}^{m}\sum_{r=1}^{n}\sum_{\substack{l_1,l_2,\cdots,l_r\\ i=l_1}}\frac{V^0}{x^0_{j,l_1}x^0_{j,l_2}\cdots x^0_{j,l_r}}\Delta x_{j,l_1}\Delta x_{j,l_2}\cdots\Delta x_{j,l_r}\ /\ r,$$

where $\Delta x_{j,l_i} = x^T_{j,l_i} - x^0_{j,l_i}$, for $i = 1, 2, \cdots, r$.

Thus to establish the proof, it suffices to show that within sub-category j, both values have the same coefficient for the terms corresponding to set R. That is, we will show that

$$\frac{V^0}{x^0_{j,l_1} x^0_{j,l_2} \cdots x^0_{j,l_r}} \Delta x_{j,l_1} \Delta x_{j,l_2} \cdots \Delta x_{j,l_r} = \sum_{\substack{R=\{l_1,l_2,\ldots,l_r\} \\ K\subseteq R}} (-1)^{r-|K|} v_j(K)$$

From Table 4-1, we know

$$\sum_{\substack{R=\{l_1,l_2,\ldots,l_r\} \\ K\subseteq R}} (-1)^{r-|K|} v_j(K) = \sum_{\substack{R=\{l_1,l_2,\ldots,l_r\} \\ K\subseteq R}} (-1)^{r-|K|} \Big(\prod_{l\in K} x^T_{j,l} \prod_{p\in N\setminus K} x^0_{j,p} - \prod_{i=1}^{n} x^0_{j,i}\Big)$$
$$= \sum_{\substack{R=\{l_1,l_2,\ldots,l_r\} \\ K\subseteq R}} (-1)^{r-|K|} \Big(\prod_{l\in K} x^T_{j,l} - \prod_{l\in K} x^0_{j,l}\Big) \prod_{p\in N\setminus K} x^0_{j,p}$$

For each subset K of R,

$$\Big(\prod_{l\in K} x^T_{j,l} - \prod_{l\in K} x^0_{j,l}\Big) \prod_{p\in N\setminus K} x^0_{j,p} = \sum_{l\in K} \Big[(x^T_{j,l} - x^0_{j,l}) \prod_{p\in N\setminus\{l\}} x^0_{j,p}\Big]$$
$$+ \sum_{l_1,l_2\in K} \Big[(x^T_{j,l_1} - x^0_{j,l_1})(x^T_{j,l_2} - x^0_{j,l_2}) \prod_{p\in N\setminus\{l_1,l_2\}} x^0_{j,p}\Big]$$
$$+ \cdots$$
$$+ \prod_{l\in K} (x^T_{j,l} - x^0_{j,l}) \prod_{p\in N\setminus K} x^0_{j,p}.$$

we find its sum of coefficients is

$\sum_{k=1}^{r} (-1)^{r-k} \binom{r-1}{k-1} = \sum_{k=1}^{r} (-1)^{(r-1)-(k-1)} \binom{r-1}{k-1} = (1-1)^{r-1} = 0.$ Similarly for any fixed proper subset L of R, the sum of coefficients corresponding to the term

$\prod_{l\in L\subset R, L\neq R} (x^T_{j,l} - x^0_{j,l}) \prod_{p\in N\setminus L} x^0_{j,p}$ is

$\sum_{k=|L|}^{r} (-1)^{r-k} \binom{r-|L|}{k-|L|} = \sum_{k=|L|}^{r} (-1)^{(r-|L|)-(k-|L|)} \binom{r-|L|}{k-|L|} = (1-1)^{r-|L|} = 0.$ Now considering the terms corresponding to $\prod_{l\in R}(x_{j,l}^T - x_{j,l}^0)\prod_{p\in N\setminus R} x_{j,p}^0$, we find that there is only one of them and its coefficient is $(-1)^{r-r} = 1$. Hence,

$$\sum_{\substack{R=\{l_1,l_2,\cdots,l_r\}\\ K\subseteq R}} (-1)^{r-|K|} v_j(K) = \prod_{l\in R}(x_{j,l}^T - x_{j,l}^0)\prod_{p\in N\setminus R} x_{j,p}^0$$

$$= \frac{V^0}{x_{j,l_1}^0 x_{j,l_2}^0 \cdots x_{j,l_r}^0}\Delta x_{j,l_1}\Delta x_{j,l_2}\cdots\Delta x_{j,l_r}.$$

And the proof is done.

Shapley values with characteristic functions in the Laspeyres index and Paasche index forms are the same

A simplified way of expressing the Shapley value (Shapley, 1953) is

$$\phi_i(v) = \sum_{i\in S\subseteq N, |S|=s} \frac{(s-1)!(n-s)!}{n!}[v(S) - v(S-\{i\})].$$

Using this expression, we will show that the Shapley values with characteristic functions in the Laspeyres index and Paasche index forms as shown in Table 4-1 are the same. First for the Laspeyres case, we have

$$\phi_i(v) = \sum_{i\in S\subseteq N, |S|=s} \frac{(s-1)!(n-s)!}{n!}[v(S) - v(S-\{i\})]$$

$$= \sum_{i\in S\subseteq N, |S|=s} \frac{(s-1)!(n-s)!}{n!}\sum_{j=1}^{m}\left(\prod_{l\in S} x_{j,l}^T \prod_{p\in N\setminus S} x_{j,p}^0 - \prod_{l\in(S-\{i\})} x_{j,l}^T \prod_{p\in N\setminus(S-\{i\})} x_{j,p}^0\right).$$

Next, for the Paasche case, we have

$$\phi_i(v) = \sum_{N\backslash(S-\{i\})\subseteq N,|S|=s} \frac{((n-s+1)-1)!(n-(n-s+1))!}{n!}[v(N\setminus(S-\{i\}))-v(N\setminus S)]$$

$$= \sum_{N\backslash(S-\{i\})\subseteq N,|S|=s} \frac{(s-1)!(n-s)!}{n!}\sum_{j=1}^{m}(-\prod_{l\in N\backslash(S-\{i\})} x_{j,l}^{0} \prod_{p\in(S-\{i\})} x_{j,p}^{T} + \prod_{l\in N\backslash S} x_{j,l}^{0} \prod_{p\in S} x_{j,p}^{T}) \quad (36)$$

$$= \sum_{N\backslash(S-\{i\})\subseteq N,|S|=s} \frac{(s-1)!(n-s)!}{n!}\sum_{j=1}^{m}(\prod_{p\in S} x_{j,p}^{T} \prod_{l\in N\backslash S} x_{j,l}^{0} - \prod_{p\in(S-\{i\})} x_{j,p}^{T} \prod_{l\in N\backslash(S-\{i\})} x_{j,l}^{0}).$$

We see that the Shapley values for the two cases are the same.

Shapley decomposition for the general case

The Shapley value with the characteristic function given in the general form in Table 4-1, can be expressed as

$$\begin{aligned}\phi_i(v)^{general} &= \sum_{i\in S\subseteq N,|S|=s} \frac{(s-1)!(n-s)!}{n!}[v(S)-v(S-\{i\})] \\ &= (1-\alpha)\cdot\phi_i(v)^{Laspeyres} + \alpha\cdot\phi_i(v)^{Paasche} \\ &= \phi_i(v)^{Laspeyres} = \text{Decomposition value from the S/S method.}\end{aligned}$$

The decomposition results for the S/S method are therefore the same as those given by the Shapley decomposition for the general case.

Appendix B: Energy Consumption and Activity Data for US Manufacturing Sector

Table B-1 to B-3 give the data used in numerical examples in Chapter 6. The data are taken from the US department of EERE online.

Table B-1: Energy consumption and activity for US 'Wood Product Manufacturing' sub-sector, 1994-2004

	Value Added $Million ($2000)	Delivered Energy (TBtu)	Energy Intensity Btu/$
1994	27,501	499.1	18.1
1995	30,378	496.1	16.3
1996	29,811	477.9	16.0
1997	29,266	485	16.6
1998	29,927	512	17.1
1999	30,448	457.4	15.0
2000	31,437	402.7	12.8
2001	30,889	393.45	12.7
2002	30,324	379.8	12.5
2003	31,374	431.1	13.7
2004	32,380	490.7	15.2

Table B-2: Energy consumption for US manufacturing sector, 1990 to 2004 (TBtu)

Sector / Year	311/312	313/314	315/316	321	322	323	324	325	326	327	331	332	333	334	335	336	337	339	Total
1990	924	257	55	425	2642	98	3052	3093	246	1022	1651	340	222	181	165	351	76	51	14848
1991	975	262	57	460	2524	97	3203	2995	242	901	1514	338	223	180	151	350	72	49	14593
1992	985	281	96	427	2671	99	2980	3086	253	980	1499	326	209	184	164	366	70	47	14723
1993	1237	292	83	483	2726	96	3573	3145	267	1030	1743	405	216	181	193	411	74	55	16208
1994	1226	304	77	499	2688	102	3284	3038	282	941	1797	406	228	182	194	393	70	56	15765
1995	1228	298	82	496	2594	109	3062	3033	296	954	1730	430	239	185	216	394	75	61	15481
1996	1084	261	72	478	2659	96	3141	3534	301	902	1820	399	212	208	204	482	86	81	16022
1997	1143	269	65	485	2747	93	3264	3416	310	987	1784	389	207	201	174	485	94	80	16192
1998	1151	279	63	512	2754	98	3625	3346	316	977	1895	416	212	196	143	473	88	84	16626
1999	1179	281	57	457	2734	100	3646	3793	338	1035	1803	445	215	192	145	466	89	83	17057
2000	1142	242	50	403	2529	92	3763	3828	337	984	1749	406	198	195	139	440	90	77	16662
2001	1317	272	45	393	2495	97	3070	3098	356	1050	1570	378	192	181	194	407	69	69	15255
2002	1230	260	37	380	2383	99	3178	3201	355	1057	1572	366	176	183	173	437	66	72	15223
2003	1138	234	30	431	2431	91	3452	2992	341	993	1480	327	153	158	159	366	61	65	14900
2004	1137	231	32	491	2583	100	3583	2951	377	993	1737	344	171	147	157	387	68	70	15556

Table B-3: Activity for US manufacturing sector, 1990 to 2004 (Million 2000$)

Sector / Year	311/312	313/314	315/316	321	322	323	324	325	326	327	331	332	333	334	335	336	337	339	Total
1990	140016	22947	30537	34221	55156	48635	16307	132099	39239	32728	37182	91859	105820	16475	42534	156900	24447	40494	1067595
1991	140059	23039	31021	31553	57243	47077	17246	129365	41162	29824	37821	84781	92726	17586	40598	157844	22815	40462	1042223
1992	143405	25382	31998	28686	60810	49531	19297	132437	44139	33729	39192	86641	91028	19400	41284	155617	24538	40627	1067743
1993	143659	26222	31588	25769	66884	47054	26109	134100	48750	33711	42014	90033	91260	22061	43715	161010	26429	41251	1101620
1994	153642	27545	32138	27501	69819	49430	24607	145232	52756	37620	43320	102878	96430	27160	47067	165376	26838	42430	1171787
1995	174714	27417	31489	30378	56785	48682	20837	141578	52299	38420	41974	107970	103950	37955	46369	160444	27201	44738	1193201
1996	162526	26545	29363	29811	60609	48172	27038	145062	56220	38150	43788	110589	99179	50212	44527	160924	27214	48386	1208316
1997	156598	26950	27999	29266	63939	47262	30396	152008	60694	43758	45212	113499	102579	66362	46516	165959	29256	49408	1257663
1998	153108	26262	26378	29927	60013	47738	36471	149755	62423	44149	45894	114384	113800	96265	44573	179915	30208	50388	1311651
1999	155058	25560	24392	30448	61016	48489	33470	157096	64661	45121	48131	114862	104960	125407	48017	181933	31487	52056	1352164
2000	154809	26453	25052	31437	55594	49009	26248	157057	66728	45743	48193	121686	109296	185563	50580	182544	32712	57515	1426219
2001	156012	21548	22716	30889	48785	45272	23939	153090	61420	45171	43172	109444	100403	181894	48495	169996	29078	55252	1346575
2002	153684	21375	21120	30324	50835	43522	32494	170484	62874	45531	44123	104382	93306	185756	48839	190899	29192	56384	1385123
2003	153281	23057	18707	31374	48852	42538	26091	172891	64042	46613	42633	107485	92256	215003	49924	198370	28872	59604	1421592
2004	155806	23214	19714	32380	53466	44445	24691	173559	70791	48999	46471	110742	100732	260286	49285	194890	30993	66337	1506801

Appendix C: Multiplicative Decomposition Results for US Manufacturing Sector, 1990-2004

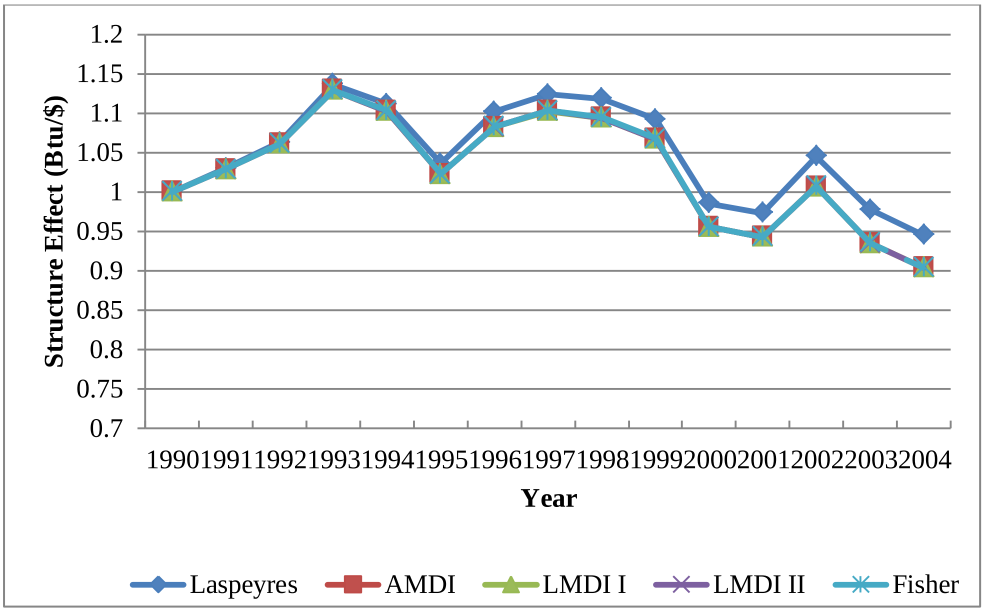

Figure C1. Decomposition results for US manufacturing sector, 1990-2004: structure effect, chaining (multiplicative decomposition).

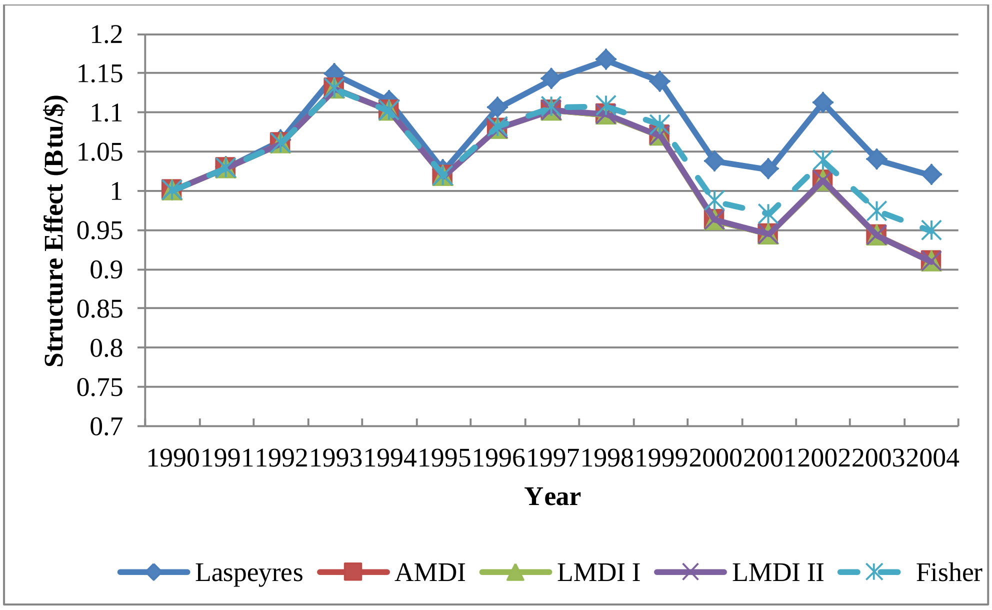

Figure C2. Decomposition results for US manufacturing sector, 1990-2004: structure effect, non-chaining (multiplicative decomposition).

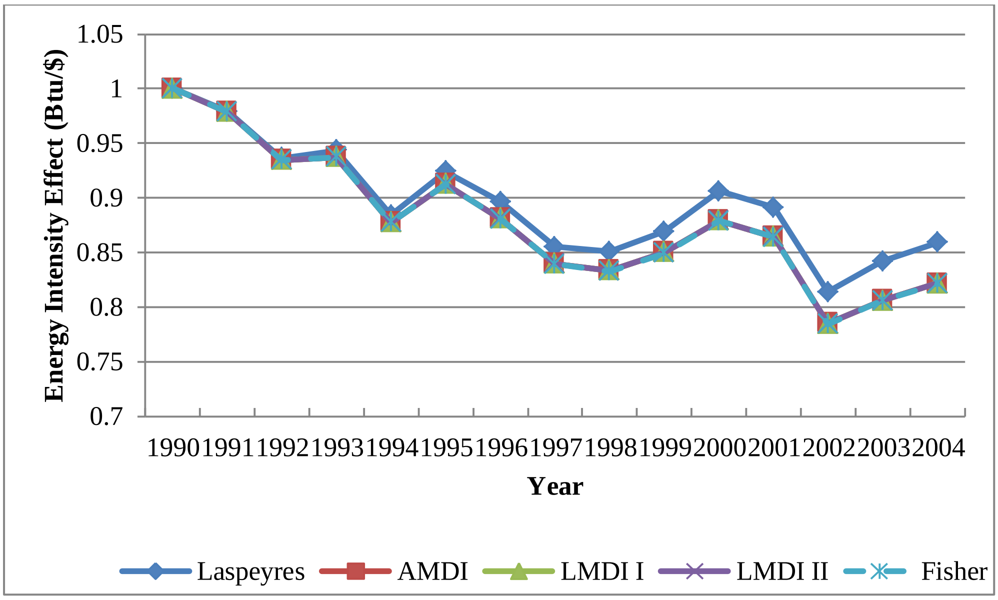

Figure C3. Decomposition results for US manufacturing sector, 1990-2004: energy intensity effect, chaining (multiplicative decomposition).

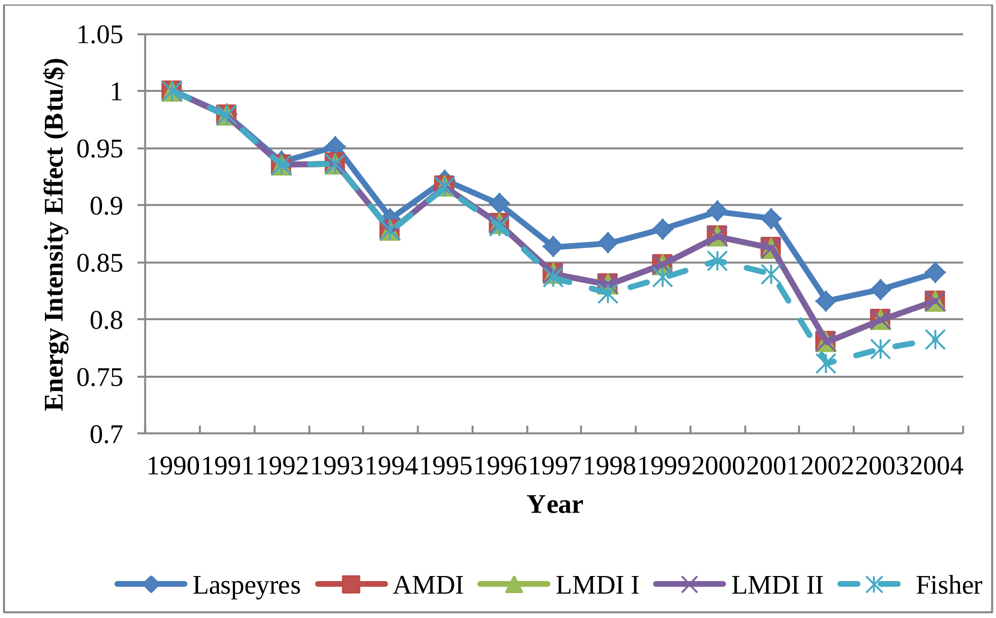

Figure C4. Decomposition results for US manufacturing sector, 1990-2004: energy intensity effect, non-chaining (multiplicative decomposition).

Appendix D: Consistency in Aggregation for Chaining Approach

To illustrate consistency in aggregation issues for the chaining approach, we use an aggregation structure of two levels: the sectoral level and the sub-sectoral level. At the sectoral level, subscript *i* refers to the first level of disaggregation which has a total of p sectors. In the sub-sectoral level, subscript *j* refers to the second level of disaggregation which has total of l_i sub-sectors in sector *i*.

In the Laspeyres index, $\Delta V_{x_k}^{0,T}$ from chaining one-step analysis is given by:

$$\Delta V_{x_k}^{one-step} = \sum_{t=0}^{T-1} \Delta V_{x_k}^{t,t+1} = \sum_{t=0}^{T-1} \sum_{i=1}^{p} \sum_{j=1}^{l_i} [x_{1,ij}^t x_{2,ij}^t \ldots (x_{k,ij}^{t+1} - x_{k,ij}^t) \ldots x_{n,ij}^t]$$

When we calculate the same effect in two steps, we first calculate the value for each sector (cumulating the consecutive years in the first step):

$$\Delta V_{x_k,i}^{two-step\,(1)} = \sum_{t=0}^{T-1} \sum_{j=1}^{l_i} \left[x_{1,ij}^t x_{2,ij}^t \ldots (x_{k,lj}^{t+1} - x_{k,ij}^t) \ldots x_{n,ij}^t\right]$$

and then calculate the aggregate indicator:

$$\Delta V_{x_k}^{two-step\,(2)} = \sum_{i=1}^{p} \left[\Delta V_{x_k,i}^{two-step\,(1)}\right] = \sum_{i=1}^{p} \sum_{t=0}^{T-1} \sum_{j=1}^{l_i} \left[x_{1,ij}^t x_{2,ij}^t \ldots (x_{k,lj}^{t+1} - x_{k,ij}^t) \ldots x_{n,ij}^t\right]$$
$$= \Delta V_{x_k}^{one-step}$$

If we cumulate the consecutive years in the second step, then

$$\Delta V_{x_k,i}^{t,t+1,two-step\,(1)} = \sum_{j=1}^{l_i}\left[x_{1,ij}^{t} x_{2,ij}^{t} \ldots (x_{k,lj}^{t+1} - x_{k,ij}^{t}) \ldots x_{n,ij}^{t}\right]$$

$$\Delta V_{x_k}^{two-step\,(2)} = \sum_{t=0}^{T-1}\sum_{i=1}^{p}\left[\Delta V_{x_k,i}^{t,t+1,wo-step\,(1)}\right]$$

$$= \sum_{t=0}^{T-1}\sum_{i=1}^{p}\left[\sum_{j=1}^{l_i}\left[x_{1,ij}^{t} x_{2,ij}^{t} \ldots (x_{k,lj}^{t+1} - x_{k,ij}^{t}) \ldots x_{n,ij}^{t}\right]\right] = \Delta V_{x_k}^{one-step}$$

Thus, using the chaining approach, the Laspeyres index is consistent in aggregation in the additive measure no matter we cumulate the decomposition results of consecutive years over time in the first or second step.

Similarly, using $\Delta V_{x_k}^{two-step\,(2)} = \sum_{i=1}^{p} \Delta V_{x_k,i}^{two-step(1)} = \Delta V_{x_k}^{one-step}$, we conclude that Paasche, S/S, AMDI, and LMDI I are all consistent in aggregation in the additive measure.

In the Laspeyres index, $D_{x_k}^{0,T}$ from chaining one-step analysis is given by:

$$D_{x_z}^{one-step} = \prod_{t=0}^{T-1} D_{x_k}^{t,t+1} = \prod_{t=0}^{T-1} \frac{\sum_{i=1}^{p}\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{z,ij}^{t+1} \ldots x_{r,ij}^{t}\right)}{\sum_{i=1}^{p}\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{r,ij}^{t}\right)}$$

When we calculate the same effect in two steps, we first calculate the value for each sector (cumulating the consecutive years at the first step):

$$D_{x_z,i}^{two-step\,(1)} = \prod_{t=0}^{T-1} \frac{\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{z,ij}^{t+1} \ldots x_{r,ij}^{t}\right)}{\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{r,ij}^{t}\right)}$$

and then calculate the aggregate indicator:

$$D_{x_z,i}^{two-step\,(2)} = \sum_{i=1}^{p}\frac{V_i^0}{V^0} D_{x_z,i}^{two-step\,(1)} = \sum_{i=1}^{p}\frac{V_i^0}{V^0} \prod_{t=0}^{T-1} \frac{\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{z,ij}^{t+1} \ldots x_{r,ij}^{t}\right)}{\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{r,ij}^{t}\right)}$$

$$\neq \prod_{t=0}^{T-1} \frac{\sum_{i=1}^{p}\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{z,ij}^{t+1} \ldots x_{r,ij}^{t}\right)}{\sum_{i=1}^{p}\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{r,ij}^{t}\right)}$$

If we cumulate the consecutive years in the second step, then

$$D_{x_z,i}^{two-step\,(2)} = \prod_{t=0}^{T-1}\sum_{i=1}^{p}\frac{V_i^t}{V^t} D_{x_z,i}^{two-step\,(1)}$$

$$= \prod_{t=0}^{T-1}\sum_{i=1}^{p}\frac{V_i^t}{V^t} \frac{\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{z,ij}^{t+1} \ldots x_{r,ij}^{t}\right)}{\sum_{j=1}^{l_i}\left(x_{1,ij}^{t} x_{2,ij}^{t} \ldots x_{r,ij}^{t}\right)} = D_{x_z,i}^{one-step}$$